



DESIGN OF AN INDUSTRIAL FLEXIBLE ROBOT CONTROLLER USING MATLAB

ATEF A. ATA^{*}, ALI R. SHAHIN^{**}, SHIHAB S. ASFOUR^{*}

^{*} Dept. of Industrial Engineering., University of Miami, Coral Gables, FL. 33124.

^{**} Dept. of Mechanical Engineering., University of Miami, Coral Gables, FL. 33124.

ABSTRACT

The main objective of this work is to investigate a causal inverse dynamics of a flexible single hub-arm system with a variable tip mass and to control the end point motion. The joint trajectory was assumed and the end point trajectory was obtained directly through the solution of the inverse dynamic problem. Although the flexible link is nonminimum phase in nature, the use of feedforward torque with end point acceleration feedback gives very good performance specially for variable tip mass. A robust controller was designed for tracking the desired trajectory based on classical control methodologies.

INTRODUCTION

For the last two decades, the problem of determining the suitable force or driving torque was of great interest to scientists and engineers. This suitable force or torque must be applied at the actuator joint, to move the tip mass of a robot arm through a prescribed trajectory as fast as possible and robust for external disturbances. Currently, the trend is towards using faster flexible manipulators and larger payloads. The first attempt in solving the dynamics of the flexible robot was the formulation of the problem relative to a nominal path coordinate system by Song and Haug (1984). Naganathan and Soni (1987) obtained the solution of the inverse kinematics of the rigid link motion and then coupled the elastic deformation to it. Cannon and Schmitz (1984) introduced a modal description of a single link flexible arm and a control strategy based on optimal control theory. Bayo (1987) introduced a finite element approach for the solution of the inverse dynamics of a single link and multi link flexible manipulators. Kwon and Book (1994) presented a technique to overcome the nonminimum phase nature of the flexible link for tracking control. Ata *et al.* (1996) applied Liapunov second method to calculate the drive torque of a flexible hub-arm system through the solution of the inverse dynamics problem. The main objective of this study is to design a robust controller for a flexible hub-arm system with variable tip mass using classical control techniques.

PROBLEM FORMULATION

Consider a flexible robot arm of length (L), cross sectional area (A), density (ρ) and a flexural stiffness (EI). The link is connected to a rigid hub of radius (r) and Inertia (I_h) at one end and a tip mass (M_p) with moment of inertia (I_p) is attached at the free end as shown in Figure 1. The beam is modeled using Euler-Bernoulli's assumption and the motion is assumed to be in the horizontal plane. The equation of motion can be derived using the extended Hamilton principle

(Meirovitch, 1967). The detailed discussion of the derivation of the equations of motion of this model is given in Ata *et al.* (1996). Let the trajectory of the actuator joint be:

$$\theta(t) = 0.14 * (\pi - \sin(\pi t)) \quad (1)$$

Then the trajectory of the tip end can be obtained. Applying the assumed mode method (Meirovitch, 1967) one can assume the solution in the form:

$$W(x, t) = \psi(x)q(t) \quad (2)$$

The system equations of motion become:

$$EI \frac{\partial^4 \psi}{\partial x^4} q(t) + \rho A \psi \frac{d^2 q}{dt^2} + \rho A x \frac{d^2 \theta}{dt^2} = 0 \quad (3)$$

$$I_t \left[\frac{d^2 \theta}{dt^2} \right] M_{tp}(r+L) \left[\psi \frac{d^2 q}{dt^2} \right] \Big|_{x=r+L} + \int_r^{r+L} \rho A x \psi \left[\frac{d^2 q}{dt^2} \right] dx = T(t) \quad (4)$$

Multiply equations (3) by $\psi(x)$ and integrate over the length of the link, one can get:

$$M \frac{d^2 q}{dt^2} + Kq(t) + I \frac{d^2 \theta}{dt^2} = 0 \quad (5)$$

In which the matrices M, K, and I can be given by

$$M = \int_r^{r+L} \rho A \psi^2(x) dx, K = \int_r^{r+L} EI \psi(x) \frac{d^4 \psi(x)}{dx^4} dx, I = \int_r^{r+L} \rho A x \psi(x) dx \text{ and}$$

$$I_t = I_h + I_b + I_{tip} + I_{tp}$$

Equations 3 and 4 which describe the motion of the system can be written in a state space form as

$$\frac{dx}{dt} = Ax + Bu \quad \text{and} \quad Y = Cx + Du \quad \text{Where:}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\Omega_n^2 & -2\zeta\Omega_n & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{\gamma_2}{\gamma_3} & 0 & 0 & 0 \\ \gamma_3 & & & \end{bmatrix}, B = \begin{bmatrix} 0 \\ -I \\ M\gamma_3 \\ 0 \\ I \\ \gamma_3 \end{bmatrix}, C = [\psi(x) \quad 0 \quad (r+L) \quad 0] \text{ and } D = [0]$$

$$\Omega_n^2 = -\left\{ \frac{K}{M} + \frac{I\gamma_2}{\gamma_3} \right\}, \gamma_0 = M_{tp}(r+L)\psi(r+L) + I, \gamma_1 = \left(\frac{\gamma_0 I}{M} \right), \gamma_2 = \left(\frac{\gamma_0 K}{M} \right) \text{ and } \gamma_3 = (I_t - \gamma_1)$$

The data proposed by Bayo (1987) was used to validate the model suggested in this paper.

Table I Properties of the Flexible Link [Bayo, 1987]

Properties	Values
Length (L)	1.27 m
Cross sectional area (A)	8.066*10 ⁻⁵ m ²
Moment of area (I)	6.775*10 ⁻¹¹ m ⁴
Young modulus (E)	7.11*10 ¹⁰ N/m ²
Mass density (ρ)	2715 Kg/m ³
Tip mass (M _{tp})	(0.2-0.3) * Beam Mass Kg

CONTROLLER DESIGN

Controlling a flexible robot link with an uncertain tip mass to track a desired trajectory required addressing several issues. The controller must be robust, that is, it must be able to track the

desired trajectory for unknown changes that might happen in the tip mass. The controller must also take into account the uncertainties in the model that stems from throwing away information provided by natural frequencies higher than the fundamental natural frequency. The controller must be stable and track the desired trajectory with great accuracy, therefore a closed loop configuration must be used. There are many different robust control methodologies used in controlling a flexible link robot such as Quantitative Feedback Theory and H-Infinity. However in this paper, only basic classical control methodologies are used to design a controller by using the MATLAB software to determine uncertainty ranges and obtain Bode and Nichols plots for the family of plants. The system is nonminimum phase and the bandwidth is limited by the location of the zero (Doyle *et al.* 1992). Hence, the response of the system can be improved by using an inner-outer loop structure (Khorrami and Zheng 1992) or inverse dynamics method (Kwon and Book, 1994). In this paper it is assumed that the tip mass can change from 20% to 30% of the beam mass, but the manner in which it changes is unknown. Since the plant has a zero in the right half plane, the control methodology can be shown more clearly using a simple example. Let the tip mass be as shown below in Figure 3. The Bode plot for the family of plant transfer functions is shown in Figure 4.

A controller was designed using a tip mass nominal value of 24.1% of the beam mass. Since the plant is changing, the tracking properties can be improved by using a feedback controller. Although not impossible to handle, the stability problems caused by having two poles at the origin and a nonminimum phase zero can be eliminated by using acceleration feedback. It can be determined that close to 5 degrees of phase margin will guarantee stability for the family of transfer functions. By loop shaping, the feedback controller designed to improve the tracking and increase the bandwidth was determined to be a series of cascaded lead controllers shown below:

$$G(s) = 4 \left(\frac{(s+0.5)(s+1.5)(s+3)}{(s+1)(s+2)(s+4)} \right)$$

Figures 5 and 6 show the Nichols and Bode plots of the compensated nominal transfer function. For tracking, a two degree of freedom control methodology must be used. Some of these methodologies are Classical Quantitative Feedback Theory and inverse dynamic method (Kwon and Book, 1994). In this paper, the designed feedback controller shown above is used in conjunction with an inverse dynamics calculated torque feedforward term. The configuration and the response can be seen in Figures 7&8. It must also be mentioned that the "robot arm" S-Function the SIMULINK diagram (Fig. 7) in the transfer function relating the input torque to the tip mass acceleration. By examining Figure 8, it can be concluded that the flexible robot link tracks the desired trajectory very well.

CONCLUSIONS

The number of applications requiring robots in manufacturing is expanding everyday. For many applications, the rigid link and constant tip mass are not valid assumptions. For designing controllers, accurate state space models are required. The derivation for modeling a flexible robot arm with variable tip mass is shown in this paper. The controller must be robust to the changes in the tip mass. The controller was determined to be a closed loop (necessary for robustness due to changes in the tip mass) with a feedforward term (to reduce the error due to the nonminimum phase nature of the system). This controller gives tracking without overshoot or any residual vibrations.

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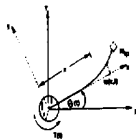


Fig. 1. Hub-Arm system.

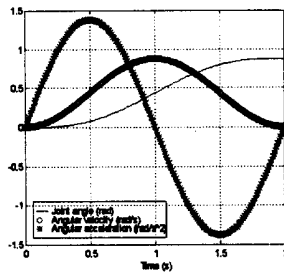


Fig. 2. Desired joint trajectory.

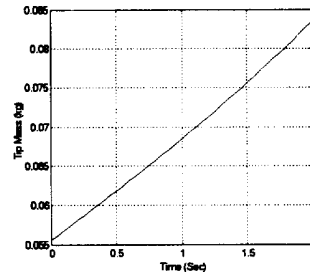


Fig. 3. Change of tip mass.

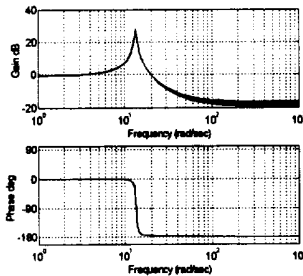


Fig. 4 . Bode plot of the original plant.

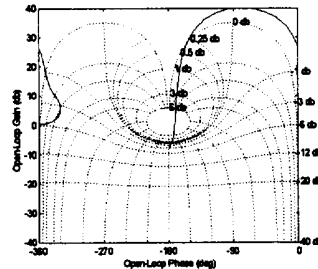


Fig. 5. Compensated Nichols plot.

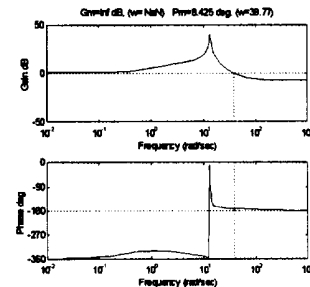


Fig. 6. Compensated Bode plot.

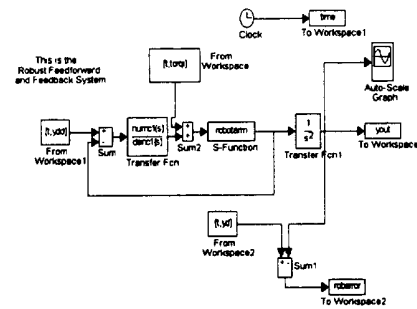


Fig. 7. Robust feedforward with feedback block diagram

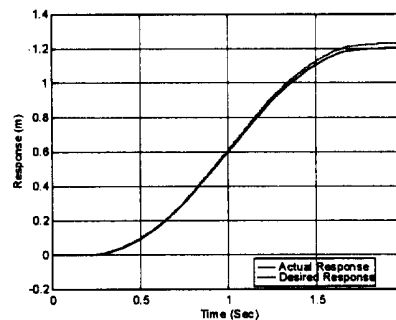


Fig. 8. Robust feedforward with Feedback performance.