多層透明被覆材料太陽能輻射性質之理論探討：I. 使用光跡追蹤法

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摘 要

本研究使用光跡追蹤法導出一系列的方程式可用來計算單、雙及多層透明被覆材料之透光率，吸收率及反射率。此些基本的太陽能輻射性質在太陽能與溫室被覆材料的研究上頗爲重要。本研究導出之單層材料透光率之計算公式與前人推導者相同。雙層材料方面，本研究分兩方向進行。其一、有空氣在其間的雙層透明被覆材料：曾有部份學者有過類似的研究，但他們的方程式有著很大的限制，使得其可用性大爲減低。其二、完全緊密接合的雙層透明被覆材料：在文獻中從未發現有類似研究發表。又，有關各種材料之衰減係數值方面，由於一般文獻中相關的記載很少且製造廠商也未能提供基本資料，實在有進一步研究之必要。本研究亦同時導出可用來求各種材料衰減係數之計算公式。

關鍵詞：透明被覆材料、太陽能輻射性質、光跡追蹤。

THEORETICAL INVESTIGATION OF SOLAR RADIATION PROPERTIES OF MULTI-LAYER GLAZINGS — PART I. USING RAY TRACING TECHNIQUE

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ABSTRACT

Ray-tracing technique was utilized to derive the equations for the calculation of the transmissivity, absorptivity and reflectivity of single, double and multiple layer (s) of glazing. The transmissivity equation for the single layer glazing that derived in this study was the same with the one derived by Robbins and Spillman. In double-layer glazing, the studies involved both the air separated double-layer and the closely connected double-layer. The later topic was never studied by other researchers for there is no similar research presented in the literature. The former topic was studied by other researchers yet they derived equations with very strict constraints which limit the usefulness of the equations. The equations derived in this study are considered more applicable and theoretically more accurate. Due to the lack of the information of the extinction coefficient of glazings from both literature and manufactures, an equation for the calculation of this important property of glazings was also developed.

Keywords: Glazing, Solar radiation property, Ray tracing

INTRODUCTION

The ray-tracing technique has been widely used by researchers to derive solar radiation properties of covers for greenhouse glazing (cladding) and for solar collectors. Using this technique Whillier (1967) derived a general equation to calculate the transmissivity of n layer(s) of glazings with same refractive index, where n can be equal to 1 or more. Assuming glazings are identical in thickness and has same extinction coefficient values, Whillier's equation can be further simplified. Based on same technique, Robbins and Spillman (1980) proposed an equation for n layers of identical glazing (same refractive index, same extinction coefficient and same thickness), yet it is different from the simplified form of Whillier's. Besides the inconsistence, both equations are subject to materials with same refractive index.

The inconsistence not only exists in multilayer equations but also can be found for single layer equations. Assuming number of layer is 1, both equations can be used to predict transmissivity of single layer glazing, yet two simplified equations are different. Both equations were widely used by other researchers. Duffie and Beckman (1974, 1980), Whillier (1977), and Garzoli (1984) utilized Whillier's equations in their research while Siegel and Howell (1981), Critten (1983, 1984), Takakura (1989), Kurata (1990) and He et al. (1990) utilized Robbins and Spillman's equations in their models.

OBJECTIVE

The objectives of this study were to

1. evaluate the equations for single and multiple layer(s) of identical glazing materials derived by former researchers,
2. develop equations for the calculation of optical properties of multi-layer glazing materials allowing any combinations of glazing materials.
3. derive equations for the calculation of optical properties of closely connected layers,
4. derive equation for the calculation of extinction coefficient assuming the direct normal transmissivity is given.

LITERATURE REVIEW

The studies of properties of transparent coverings were popular during the energy crisis. The applications of solar energy were booming then, it is still an important research topic due to the ongoing solar energy applications, booming of greenhouse industries and the advancement of the new technologies which make new glazing materials possible.

Transparent covers for greenhouse glazing and for solar collectors should be able to withstand severe environmental hazards, such as the impact of hailstones and high winds and of course should be constructed of materials which possess a high transmissivity and low reflectivity for solar radiation. The solar radiation properties of the glazing materials such as absorptivity, reflectivity and transmissivity can be determined by the surface reflectance, single pass absorptance, and the thickness of glazing. The surface reflectance can be categorized into two types: specular reflectance and nonspecular reflectance. When an incoming beam hit a smooth surface (glass, PE), the angle of the reflected beam can be determined by the incident angle, then the surface reflectance is dominated by specular reflectance. When the surface is rough (fiber glass), the fraction of incident light that is reflected such that the angle of the reflected beam is not defined by the incident angle, the surface reflectance of the material is then dominated by the nonspecular reflectance. Specular reflectance (ρ_s) is determined from the indexes of refraction, incident angle, and the polarization of the incident radiation. For unpolarized light, based on Fresnel's equation:

$$\rho_s = \frac{1}{2}\left[\frac{\sin^2(\theta_1 - \theta_2)}{\sin^2(\theta_1 + \theta_2)} + \frac{\tan^2(\theta_1 - \theta_2)}{\tan^2(\theta_1 + \theta_2)}\right] \quad (1)$$

where,

θ_1: incident angle

θ_2: refractive angle, can be determined from Snell's Law as shown in equation 2.

$$n_1 \cdot \sin(\theta_1) = n_2 \cdot \sin(\theta_2) \quad (2)$$

At normal incidence, equation 1 can be simplified as shown below:

$$\rho_s = \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2} \quad (3)$$

where,

n_1: refractive index of medium 1

n_2: refractive index of medium 2

Nonspecular reflectance (ρ_{ns}) can be approximated by equation 1 assuming θ_1 is always equal to 60 degree as suggested by Duffie and Beckman (1974), i. e., the value of the nonspecular reflectance is a constant.

MULTIPLE LAYERS SEPARATED WITH AIR

Although the so called double layer and n layers were never defined, they meant two (or n) air separated layers. Some of the glazing materials involved multiple layers with no air separated in between such as fiberglass coated with Tedlar and PE or glass with water film. There are no equations available to calculate the optical properties of previously mentioned closely connected layers.

Whillier (1967) derived an equation to calculate the transmissivity for solar radiation of n plate covers (layers) having same refractive index. The equation is as follows:

$$\tau_{1,2,\ldots,n} = \frac{1 - \rho_s}{1 + (2n - 1)\rho_s} e^{-(K_1L_1 + K_2L_2 + \cdots\cdots + K_nL_n)} \quad (4)$$

where,

K_n: extinction coefficient of layer n.

L_n: path length of light beam passing through layer n

For n layers of identical glazing materials (same refractive index, extinction coefficient, and thickness), equation 4 can be simplified as shown in equation 4.1.

$$\tau_{1,2,\ldots,n} = \frac{1 - \rho_s}{1 + (2n - 1)\rho_s} e^{-(nKL)} \quad (4.1)$$

Based on equation 4, the transmissivity for single and double layer glazing (same refractive index) can be derived.

The single-layer equation:

$$\tau = [(1 - \rho_s) / (1 + \rho_s)]\ e^{-KL} \quad (5)$$

The double-layer equation (same refractive index):

$$\tau_{1,2} = [(1 - \rho_s) / (1 + 3\rho_s)]\ e^{-(K_1L_1 + K_2L_2)} \quad (6)$$

Robbins and Spillman (1980) used the same ray-tracing technique , however , a different equation to calculate the transmissivity of the glazings was derived. As shown in Figure 1, a direct beam with the intensity of unity passing through one layer of glazing is reflected by two surfaces and absorbed by the material in between. Summing all the rays transmitted through the material, the transmissivity can be derived as shown in equation 7.

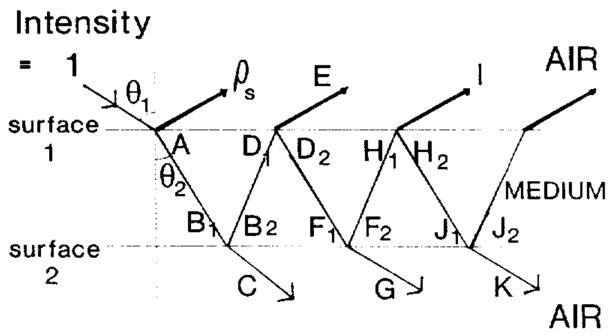

Figure 1 A schematic diagram of ray tracing technique on single layer glazing

$$\tau = \frac{(1-\rho_s)^2(1-\alpha)}{1-\rho_s^2(1-\alpha)^2} \tag{7}$$

where, α, defined as absorptance (or absorptivity) by Robbins and Spillman (1980) and many other researchers, can be derived from Bouger's Law:

$$\alpha = 1 - e^{-KL} \tag{8}$$

Following the similar analysis, Robbins and Spillman (1980) derived the compound transmissivity of double layer for identical glazings (same material, same thickness), the equation is as follows:

$$\tau_{1,2} = \frac{(1-\rho_s)(1-3\rho_s)e^{-2KL}}{1-9\rho_s^2 e^{-2KL}} \tag{9}$$

For two dissimilar glazings, an empirical equation was used:

$$\tau_{1,2} = C_\theta \tau_1 \tau_2 \tag{10}$$

where,

$$C_\theta = 0.0675\,\theta_1^2 - 0.0295\,\theta_1 + 1.010 \tag{11}$$

θ_1 : incident angle

For n layers of identical glazing:

$$\tau_{1,2,\ldots,n} = \frac{(1-\rho_s)[1-(2n-1)\rho_s]\,e^{-nKL}}{1-[(2n-1)\rho_s]^2 e^{-2KL}} \tag{12}$$

ANALYSIS

One drawback of Whillier's n layers equation is that it subject to the materials with same refractive index. The 2nd drawback is that he assumed the actual transmissivity is the product of the transmissivity due to reflection and the transmissivity due to absorption, i.e. the effect of reflection and absorption within layer were considered independent. In his approach the effect of multiple reflection within layer (between upper and lower surfaces) was considered but the effect of absorption within layer was considered occurred only once, the multiple absorption was not considered.

Same drawback occurred in Robbins and Spillman's n layers equation, the equation was subject to more constraints (same refractive index, same extinction coefficient, and same thickness). In their approaches, they considered both the multiple reflection and absorption within layer but only derived equation (equation 7) for single layer glazing and assumed absorptivity equals α (derived from equation 8) which is conceptually wrong.

The symbol α as shown in equation 7 should not be considered as the absorptivity of the glazing. The α, calculated using equation 8, if treated as the absorptivity, one might jump to the conclusion that the reflectivity (ρ) equals $1 - \tau - \alpha$ without knowing that this α was only the single pass absorptance and is not the actual absorptivity of the galzing. The α used in equations 7 and 8 should be changed to α_s and termed "single pass absorptance". The reflectivity (ρ) previously derived was merely $1 - \tau - \alpha_s$. This value is not the reflectivity of the glazing.

In deriving equations for double layer, two approaches were used by Robbins and Spillman, one for two identical glazing (equation 9) and another for two dissimilar glazing (equation 10). The effect of multiple reflections between layers was approximated using a parabolic equation as shown in equation 11 which is a function of incident angle. The drawback of this approach is that it assumed the effect of multiple reflection between layers are always the same even for different glazings which is not correct.

It is clear that two groups of researchers utilized same technique but came out with different equations (equation 4.1 vs. 12; 5 vs. 7 and 6 vs. 9). Whillier's equation (eq. 5) equals eq.7 when the term $(1-\alpha)$ in eq.7 equals 1, i.e., no absorption $(\alpha=0)$ occurred and the product of extinction coefficient and thickness equals 0 $(e^{-KL}=1)$, which does not exist in any glazings.

DEVELOPMENT OF NEW EQUATIONS

Based on the ray-tracing technique, summing all the rays that is transmitted through the glazing, the equation to calculate the transmissivity of the glazing can be derived. The equation is the same with the one derived by Robbins and Spillman as shown in equation 7. This verified that the method we use is correct. Summing all the rays that is reflected by the material (dark arrows in Figure 1), the reflectivity can be derived as shown in equation 13 (see Appendix for details).

$$\rho=\rho_s+\frac{\rho_s(1-\rho_s)^2\tau_s^2}{1-\rho_s^2\tau_s^2}=\rho_s(1+\tau\tau_s) \qquad (13)$$

where,

ρ :reflectivity of the glazing

ρ_s : specular (surface) reflectance

τ : transmissivity of the glazing (derived from eq. 7)

τ_s : single pass transmittance,

$\tau_s=e^{-KL}=1-\alpha_s$

α_s : single pass absorptance,

$\alpha_s=1-e^{-KL}=1-\tau_s$

With the concept of grouping plus the same ray-tracing technique, equations for calculating the compound transmissivity and reflectivity of any two glazing were developed as shown in equations 14 and 15., and of course the absorptivity $(\alpha_{1,2})$ equals $1-\tau_{1,2}-\rho_{1,2}$.

The idea of grouping is to treat one layer (medium) instead of one surface at a time as shown in Figure 2. The effect of multiple reflection and absorption within layer has been considered in the approach shown in Figure 1. The approach shown in Figure 2 deals with multiple reflection between layers with the assumption that there are no absorption occurred between layers. For the medium between layers is air, this assumption is acceptable.

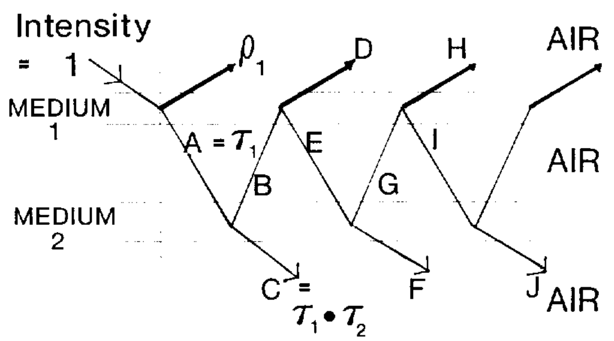

Figure 2 A schematic diagram of ray tracing technique on air separated double-layer glazing

$$\tau_{1,2}=\frac{\tau_1\tau_2}{1-\rho_1\rho_2} \qquad (14)$$

$$\rho_{1,2}=\rho+\frac{\tau_1^2\rho_2}{1-\rho_1\rho_2} \qquad (15)$$

where,

$\tau_{1,2}$: transmissivity of the double layer glazing

$\rho_{1,2}$: reflectivity of the double layer glazing

τ_1 : transmissivity of the 1st layer given by equation 7

τ_2 : transmissivity of the 2nd layer given by equation 7

ρ_1 : reflectivity of the 1st layer given by equation 13

ρ_2 : reflectivity of the 2nd layer given by equation 13

Equations 14 and 15 can be used for any combination of two layers no matter they are identical or dissimilar. This approach is more accurate than the parabolic equation used by Robbins and Spillman. Also note that the value of $\rho_{1,2}$ is different from the value of $\rho_{2,1}$ for dissimilar glazing materials (see Appendix for details, Figure 3). Equations 14 and 15 can be extended

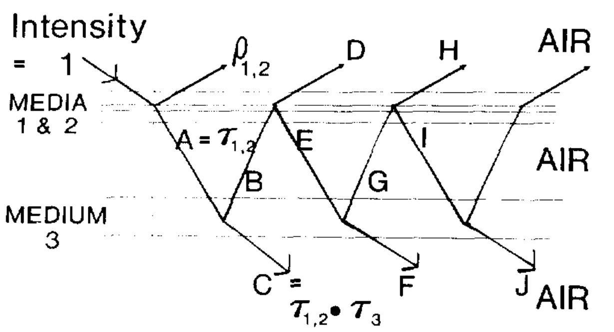

Figure 3 A schematic diagram of ray tracing technique on air separated triple-layer glazing

to any number of covers by adding one cover at a time and by giving proper attention to subscript order. For triple layer glazing, the transmissivity and reflectivity are given by:

$$\tau_{1,2,3} = \frac{\tau_{1,2}\tau_3}{1 - \rho_{1,2}\rho_3} \tag{16}$$

$$\rho_{1,2,3} = \rho_{1,2} + \frac{\tau_{1,2}^2\rho_3}{1 - \rho_{1,2}\rho_3} \tag{17}$$

where,

$\tau_{1,2,3}$: transmissivity of triple layer glazing

$\rho_{1,2,3}$: reflectivity of triple layer glazing

τ_3 : transmissivity of the 3rd layer given by eq. 7

ρ_3 : reflectivity of the 3rd layer given by eq. 13

For n layers separated with air, the transmissivity and reflectivity can be formulated as follows:

$$\tau_{1,2,\ldots,n} = \frac{\tau_{1,2,\ldots,n-1}\tau_n}{1 - \rho_{1,2,\ldots,n-1}\rho_n} \tag{18}$$

$$\rho_{1,2,\ldots,n} = \rho_{1,2,\ldots,n-1} + \frac{\tau_{1,2,\ldots,n-1}^2\rho_n}{1 - \rho_{1,2,\ldots,n-1}\rho_n} \tag{19}$$

CLOSELY CONNECTED DOUBLE LAYER (WITH NO AIR IN BETWEEN)

With a little modification on ray-tracing technique (see Appendix for details), equations for calculating the transmissivity and reflectivity of two closely connected layers were derived (Figure 4).

$$\tau_{1,2} = \frac{(1 - \rho_{sa})(1 - \rho_{s2})\,\tau_{s2}^2}{1 - \rho_{sa}\rho_{s2}\tau_{s2}^2} \tag{20}$$

$$\rho_{1,2} = \rho_{sa} + \frac{\rho_{s2}(1 - \rho_{sa})^2\tau_{s2}^2}{1 - \rho_{sa}\rho_{s2}\tau_{s2}^2} \tag{21}$$

where,

ρ_{sa}: intermediate variable

$$\rho_{sa} = \rho_{s0} + \frac{\rho_{s1}(1 - \rho_{s0})^2\tau_{s1}^2}{1 - \rho_{s0}\rho_{s1}\tau_{s1}^2} \tag{22}$$

ρ_{s0}: Specular reflectance of surface 0

ρ_{s1}: specular reflectance of surface 1

ρ_{s2}: specular reflectance of surface 2

τ_{s1}: single pass transmittance of layer 1,

$$\tau_{s1} = e^{-K_1L_1} = (1-\alpha_{s1})$$

τ_{s2}: single pass transmittance of layer 2,

$$\tau_{s2} = e^{-K_2L_2} = (1-\alpha_{s2})$$

α_{s1}: single pass absorptance of layer 1

α_{s2}: single pass absorptance of layer 2

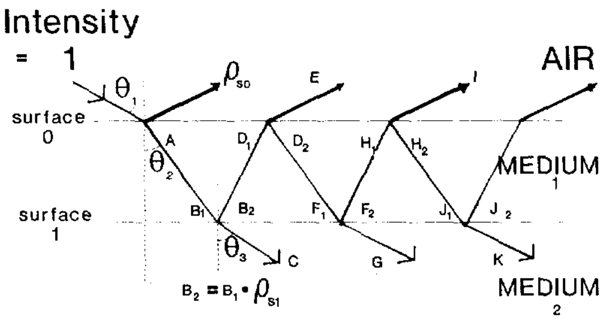

Figure 4 A schematic diagram of ray tracing technique on closely connected double-layer glazing

The intermediate variable ρ_{sa} can be considered as the apparent specular reflectance of compound surfaces 0 and 1. The two closely connected layers can be treated as a single layer with the transformation of the specular reflectance (equation 1) into apparent specular reflectance (equation 22). After the transformation, equations 20 and 21 are exactly the same with equations 7 and 13, respectively.

There exists some glazing materials casted with other material, such as fiberglass casted with polyvinyl fluoride (PVF, Tedlar) and glass casted with tinoxide, etc, to either reduce the reflectance or improve weathering. No matter what the purposes are, the specular reflectance of the original layer is altered due to the different refractive index of the casted material. Since this casted layer is very thin (within few μm), the single pass absorptance of layer 1 (outside layer) in equation 22 can be neglected ($\alpha_{s1} = 0$, i.e., $\tau_{s1} = 1$), equation 22 can be simplified as shown in equation 23.

$$\rho_{sa} = \frac{\rho_{s1}(1-\rho_{s0}) + \rho_{s0}(1-\rho_{s1})}{1-\rho_{s0}\rho_{s1}} \quad (23)$$

At normal incidence, ρ_{s0} in equation 23 can be substitute using equation 26 for one of the media is air and ρ_{s1} can be substitute using equation 3, thus, equation 23 is transformed into equation 24 as shown below.

$$\rho_{sa} = 1 - \frac{4n_1n_2}{(n_1^2+n_2)(1+n_2)} \quad (24)$$

where,

n_1: refractive index of the first (outside) layer

n_2: refractive index of the second (inside) layer

When n_1 equals the square root of n_2, the specular reflectance of the compound layer can be minimized. For example, the specular reflectance of glass (n = 1.526) at normal incidence is 0.0434, if coated with material (n = 1.23), the specular reflectance of compound layer at normal incidence is reduced to 0.022.

EXTINCTION COEFFICIENT

Refractive index determines the reflection losses from the cover while the extinction coefficient determines the absorption loss in the cover. In order to calculate the transmissivity of a covering materials, the knowledge of refractive index and the extinction coefficient of that particular material is necessary. The information of the extinction coefficient is less available than the refractive index from both the literature and from the manufacturers. It is fortunately that for most common glazings, there are little change of extinction coefficient over the solar spectrum in the range of 0.3 to 3 microns (300 to 3000 nm) (Albright, 1990).

An equation to calculate extinction coefficient was developed by Garzoli (1984), however, his approach was based on Whillier's equation (equation 6), it is considered accurate only when dealing with very thin film, materials with very low extinction coefficient, or both.

If the information of specular reflectance, thickness of glazing and direct transmissivity (incident angle = 0) are available, the extinction coefficient of a single layer glazing can be derived as shown in equation 25.

$$K = -LN\ [(X^{1/2} - B)/(2\tau\rho_s^2)]/t \qquad (25)$$

where,

K : extinction coefficient of glazing, in mm^{-1}

t : thickness of glazing, in mm

τ : direct transmissivity (incident angle = 0), decimal

LN: natural log

$X = (B^2 + 4\tau^2\rho_s^2)$

$B = (\rho_s - 1)^2$

$$\rho_s = \frac{(n_1 - 1)^2}{(n_1 + 1)^2} \text{ assuming the 2nd medium is air} \qquad (26)$$

If the single layer glazing is coated (casted or cladded) with a material with different refractive index, the specular reflectance (ρ_s) used in equation 25 should be replaced by using apparent specular reflectance (ρ_{sa}), i.e. instead of using equation 26 to calculate ρ_s, one should use equation 24 to calculate ρ_{sa}.

CONCLUSION

The equations presented in this study can provide accurate prediction of transmissivity, reflectivity and absorptivity for smooth and homogeneous transparent glazings such as the materials that made of glass, polyethylene (PE), polyester, polyvinyl chloride (PVC), Lexan① (polycarbonate), Mylar (polyethylene terephthalate), or Tedlar (polyvinyl fluoride, PVF). The validation of these equations will be presented in the part II of this paper.

Equations for double and multiple layers of glazing were modified based on same ray-tracing technique with the help of the grouping concept. New equations are more applicable and theoretically more accurate than the equations developed by former researchers. New equations were derived to find the extinction coefficient of glazings and to calculate the solar radiation properties of closely connected double-layer glazings. These equations can be very useful in the study of plasticulture.

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APPENDIX

Detail descriptions of ray-tracing technique

Single Layer

As shown in Figure 1, there are 3 media (air, glazing material and air) and 2 surfaces (surfaces 1 and 2) involved in this analysis. The amount of each light intensity that either reflected by or transmitted through the surfaces can be determined based on the specular reflectance (ρ_s) of the surfaces and the single pass absorptance (α_s) of the glazings. The single pass transmittance (τ_s) is defined as equals 1 minus α_s. Assuming the incoming light has an intensity of unity, the amount of each ray are as follows:

$$
\begin{aligned}
A &= 1-\rho_s \\
B_1 &= A(1-\alpha_s) = (1-\rho_s)\tau_s \\
B_2 &= B_1\rho_s = \rho_s(1-\rho_s)\tau_s \\
C &= B_1(1-\rho_s) = (1-\rho_s)^2\tau_s \\
D_1 &= B_2(1-\alpha_s) = \rho_s(1-\rho_s)\tau_s^2 \\
D_2 &= D_1\rho_s = \rho_s^2(1-\rho_s)\tau_s^2 \\
E &= D_1(1-\rho_s) = \rho_s(1-\rho_s)^2\tau_s^2 \\
F_1 &= D_2(1-\alpha_s) = \rho_s^2(1-\rho_s)\tau_s^3 \\
F_2 &= F_1\rho_s = \rho_s^3(1-\rho_s)\tau_s^3 \\
G &= F_1(1-\rho_s) = \rho_s^2(1-\rho_s)^2\tau_s^3 \\
H_1 &= F_2(1-\alpha_s) = \rho_s^3(1-\rho_s)\tau_s^4 \\
H_2 &= H_1\rho_s = \rho_s^4(1-\rho_s)\tau_s^4 \\
I &= H_1(1-\rho_s) = \rho_s^3(1-\rho_s)^2\tau_s^4 \\
J_1 &= H_2(1-\alpha_s) = \rho_s^4(1-\rho_s)\tau_s^5 \\
J_2 &= J_1\rho_s = \rho_s^5(1-\rho_s)\tau_s^5 \\
K &= J_1(1-\rho_s) = \rho_s^4(1-\rho_s)^2\tau_s^5
\end{aligned}
$$

$$
\begin{aligned}
Transmissivity\ (\tau) &= (C+G+K+\ldots)/1 \\
&= (1-\rho_s)^2\tau_s + \rho_s^2(1-\rho_s)^2\tau_s^3 + \rho_s^4(1-\rho_s)^2\tau_s^5 + \ldots\ldots \\
&= (1-\rho_s)^2\tau_s[1+\rho_s^2\tau_s^2+\rho_s^4\tau_s^4+\ldots] \\
&= \frac{(1-\rho_s)^2\tau_s}{1-\rho_s^2\tau_s^2} = \frac{(1-\rho_s)^2e^{-KL}}{1-\rho_s^2e^{-2KL}} \qquad [eq.7]
\end{aligned}
$$

$$
\begin{aligned}
Reflectivity\ (\rho) &= (\rho_s+E+I+\ldots)/1 \\
&= \rho_s + \rho_s(1-\rho_s)^2\tau_s^2 + \rho_s^3(1-\rho_s)^2\tau_s^4 + \ldots \\
&= \rho_s + \frac{\rho_s(1-\rho_s)^2\tau_s^2}{1-\rho_s^2\tau_s^2} \\
&= \rho_s(1+\tau_s\tau) \qquad [eq.13]
\end{aligned}
$$

Absorptivity (α) = $1-\tau-\rho$

Double Layer separated with air

As shown in Figure 2, using same tracing technique but treating one layer at a time which is different from Figure 1 treating one surface at a time.

$A = \tau_1$
$B = A\rho_2 = \tau_1\rho_2$
$C = A\tau_2 = \tau_1\tau_2$
$D = B\tau_1 = \tau_1^2\rho_2$
$E = B\rho_1 = \tau_1\rho_2\rho_1$
$F = E\tau_2 = \tau_1\rho_2\rho_1\tau_2$
$G = E\rho_2 = \tau_1\rho_2^2\rho_1$
$H = G\tau_1 = \tau_1^2\rho_2^2\rho_1$
$I = G\rho_1 = \tau_1\rho_2^2\rho_1^2$
$J = I\tau_2 = \tau_1\rho_2^2\rho_1^2\tau_2$

Note that, τ_1 and τ_2 are the transmissivity of glazings 1 and 2, respectively. ρ_1 and ρ_2 are the reflectivity of glazings 1 and 2, respectively.

$$Transmissivity\ (\tau_{1,2}) = (C+F+J+\ldots)/1$$
$$= \tau_1\tau_2 + \tau_1\rho_2\rho_1\tau_2 + \tau_1\rho_2^2\rho_1^2\tau_2 + \ldots$$
$$= \tau_1\tau_2(1+\rho_1\rho_2+\rho_1^2\rho_2^2+\ldots)$$
$$= \frac{\tau_1\tau_2}{1-\rho_1\rho_2} \qquad [eq.14]$$

$$Reflectivity(\rho_{1,2}) = (\rho_1 + D + H + \ldots)/1$$
$$= \rho_1 + \tau_1^2\rho_2 + \tau_1^2\rho_2^2\rho_1 + \ldots$$
$$= \rho_1 + \tau_1^2\rho_2(1+\rho_2\rho_1+\ldots)$$
$$= \rho_1 + \frac{\tau_1^2\rho_2}{1-\rho_1\rho_2} \qquad [eq.15]$$

Absorptivity ($\alpha_{1,2}$) = $1-\tau_{1,2}-\rho_{1,2}$

Triple and More Layers separated with air

For 3 layers separated with air :

As shown in Figure 3, treating the 1st and 2nd layers (media 1 and 2) as a group, the 3rd layer (medium 3) as the 2nd group, triple layer becomes double layer, the equation for double layer can be used for triple layer with small adjustments: replace τ_1 with $\tau_{1,2}$, replace ρ_1 with $\rho_{1,2}$, and change subscript 2 to subscript 3. The results are equations 16 and 17.

$$\tau_{1,2,3} = \frac{\tau_{1,2}\tau_3}{1-\rho_{1,2}\rho_3} \qquad [eq.16]$$

$$\rho_{1,2,3} = \rho_{1,2} + \frac{\tau_{1,2}^2\rho_3}{1-\rho_{1,2}\rho_3} \qquad [eq.17]$$

For n layers separated with air :

$$\tau_{1,2,\ldots,n} = \frac{\tau_{1,2,\ldots,n-1}\tau_n}{1-\rho_{1,2,\ldots,n-1}\rho_n} \qquad [eq.18]$$

$$\rho_{1,2,\ldots,n} = \rho_{1,2,\ldots,n-1} + \frac{\tau_{1,2,\ldots,n-1}^2\rho_n}{1-\rho_{1,2,\ldots,n-1}\rho_n} \qquad [eq.19]$$

Double Layer closely connected

Sun ray passing through two closely connected layers as shown in Figure 4 involved with 3 surfaces. Let's look at two surfaces at a time. Figure 4 is similar to Figure 1 with difference in the 3rd medium (air in Figure 1 and medium 2 in Figure 4). We need to assign different ρ_s on different surfaces. The specular reflectance should be ρ_{s0} for surface 0, ρ_{s1} for surface 1, and single pass absorptance should be α_{s1} for glazing 1 (in between surfaces 0 and 1). After these modifications equations 7 and 13 can be rewritten as equations A.1 and A.2:

$$\tau_a = \frac{(1-\rho_{s0})(1-\rho_{s1})(1-\alpha_{s1})}{1-\rho_{s0}\rho_{s1}(1-\alpha_{s1})^2} \qquad [A.1]$$

$$\rho_{sa} = \rho_{s0} + \frac{\rho_{s1}(1-\rho_{s0})^2(1-\alpha_{s1})^2}{1-\rho_{s0}\rho_{s1}(1-\alpha_{s1})^2} \qquad [A.2]$$

where,

τ_a : intermediate variable
ρ_{sa} : intermediate variable
α_{s1} : single pass absorptance of glazing 1. $1-\alpha_{s1}$ equals τ_{s1} equals $e^{-K_1L_1}$

τ_{s1} : single pass transmittance of glazing 1.

The next step is to treat surfaces 0 and 1 as a group, then add surface 2. Figure 4 can be used again but replace medium 1 with media 1 and 2 and replace medium 2 with air. The new equations can be derived by replacing ρ_{s0}, ρ_{s1}, and α_{s1} in equation A.1 with ρ_{sa}, ρ_{s2}, and α_{s2}, respectively. Note that when calculating ρ_{s1} and ρ_{s2}, the incident angle is θ_1 and θ_2, respectively (Figure 4). Equations A.1 and A.2 can be rewritten as follows:

$$\tau = \frac{(1-\rho_{sa})(1-\rho_{s2})(1-\alpha_{s2})}{1-\rho_{sa}\rho_{s2}(1-\alpha_{s2})^2} \quad \text{[eq.20]}$$

$$\rho = \rho_{sa} + \frac{\rho_{s2}(1-\rho_{sa})^2(1-\alpha_{s2})^2}{1-\rho_{sa}\rho_{s2}(1-\alpha_{s2})^2} \quad \text{[eq.21]}$$

where,

τ : transmissivity of two closely connected layers

ρ : reflectivity of two closely connected layers

α_{s2} : single pass absorptance of glazing 2. $1-\alpha_{s2}$ equals τ_{s2} equals $e^{-K_2L_2}$

τ_{s2} : single pass transmittance of glazing 2.

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