CHAPTER 3
HEAT TRANSFER BASICS

3-1. Introduction

Building environment analysis and design must be based on understanding the three modes of heat transfer: conduction, convection, and radiation. Each of the three can be present independently, but usually at least two, and frequently all three, are present simultaneously and contribute significantly to determining environmental conditions inside buildings.

Heat transfer mechanisms involving phase changes will not be discussed now. They occur frequently in environmental situations, especially related to changes of thermal energy between animals and their environment, and we will return briefly to them later.

Heat transfer differs from thermodynamics in a fundamental way, even though the two topics appear similar. Classical thermodynamics is based on the concept of equilibrium, or infinitesimal deviations from equilibrium. Heat transfer arises only from non-equilibrium, specifically, from finite differences of temperature.

Heat transfer may be steady state, where temperatures and heat fluxes do not change as functions of time. Heat transfer may be steady-periodic, where conditions change with time in a regular fashion and periodically return to their starting conditions (as in response to daily cycles of outside conditions). Heat transfer may be strictly transient with neither steady-periodic nor steady-state assumptions adequate to represent the process.

Recent research has applied transient heat transfer analysis to the design of environmental control methods for large, thermally massive buildings, but for most engineering designs of agricultural buildings, steady-state analysis is adequate at least as a close approximation. This text will focus on steady-state analysis.

3-1.1. Thermal Conduction. Thermal conduction is diffusion of thermal energy through a continuous, frequently stationary medium, and depends on properties of that medium. For example, the loss of heat from the indoor surface of a building's outer wall to the outdoor surface is by conduction if the wall is solid.

In a kinetic sense, conduction heat transfer is often viewed as a cascading transfer of energy of motion among particles on an atomic level. Atoms at higher temperatures are believed to possess more kinetic energy than those at lower temperatures, energy which is transferred to less active neighbors through elastic collision in a fluid, or oscillations of atoms and transport of free electrons in a solid matrix. Heat transfer by conduction is always from regions of higher temperatures to regions of lower temperatures; the second law of thermodynamics would be violated otherwise.

Thermal conduction is the only means of heat transfer through solid, opaque objects. Conduction can also occur in fluids, but dominates overall heat transfer in fluids only in laminar flow, and in the laminar sublayer of the turbulent boundary layer adjacent to solid objects.

3-1.2. Thermal Convection. Thermal convection is a process involving fluids. The term refers loosely either to thermal energy transfer from place to place within a fluid, or between a fluid and a solid surface. In this text, the term convective heat transfer will be used to denote transfer of thermal energy between a fluid and a solid. The term convection will be used to denote transport of thermal energy through fluids by large scale eddying motions. For example, a dairy cow loses heat to the surrounding air, and the heat may be eventually lost to the outdoors by conduction through the building's wall. Transfer of the heat from the cow's pelt to her surrounding air is by convective heat transfer, transfer through the air from the vicinity of the cow to the vicinity of the wall is by convection, and transfer from the air to the wall surface is by convective heat transfer.

對流 熱傳遞

The mechanism of thermal convection which is most important in determining building environment is convective heat transfer. The transfer process involves the boundary layer. In turbulent fluid flow, eddies in the main part of the flow exchange thermal energy between the boundary layer and surrounding fluid, and penetrate within the outer regions of the boundary layer. See Figure 3-1; a turbulent boundary layer is sketched. Within the laminar sublayer of the turbulent boundary layer, thermal energy exchange reduces to thermal conduction. Ultimately, all convection between fluids and solid objects is limited by the conduction process. As fluid velocity increases and the laminar sublayer thins, the region of conduction shrinks, turbulent eddies push farther into the boundary layer, and convective heat transfer is enhanced.

Thermal convection and convective heat transfer are classified in two ways. If a fan or pump causes fluid motion, the process is termed "forced convection". If

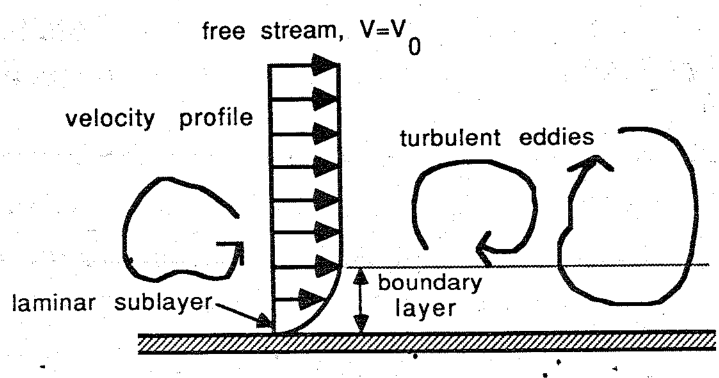


Figure 3-1. Profile of a turbulent boundary layer.

fluid motion is induced by density differences within the fluid, usually caused by temperature differences, the process is termed "natural convection", or "free convection".

3-1.3. Thermal Radiation. Thermal radiation differs markedly from both conductive and convective heat transfer processes. While conduction and convection require the presence of matter, thermal radiation heat transfer requires the absence of an intervening absorbing medium.

Radiation heat transfer occurs when electromagnetic energy leaves one object and is intercepted and absorbed by another. All objects at temperatures above absolute zero emit thermal radiation, thus radiation heat transfer is an exchange process. Objects at high temperatures emit more electromagnetic radiation than those at low temperatures; the overall process between two objects is a net transfer of thermal energy from objects at higher temperatures to those at lower temperatures. When thermal radiation leaves an object, the energy content of that object is lessened, and the object exhibits a lower temperature. The converse occurs when an object absorbs thermal radiation.

The wavelength of electromagnetic radiation traditionally considered to be involved in radiation heat transfer is the infrared band. See Figure 3-2. The lower limit of infrared radiation is at the upper limit of the visible light range, approximately 0.8 microns wavelength. The upper limit of thermal radiation is less well defined, but extends past 10 microns, a wavelength which typifies thermal radiation emitted by objects at earth temperature. However, thermal radiation exchange is not limited to these wavelength bands. Visible light and ultraviolet radiation from the sun, for examples, are electromagnetic radiation and, when absorbed, are converted to thermal energy.

also see p77.

3-2. Conduction Heat Transfer

3-2.1. The Heat Conduction Equation. The heat conduction equation for isotropic solids is derived in most heat transfer texts. One form is

$$\nabla^2 t + q_{gen} / k = (\alpha)^{-1} \delta t / \delta \tau, \qquad (3\text{-}1)$$

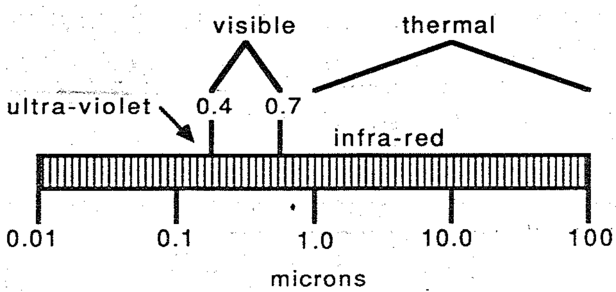


Figure 3-2. The electromagnetic radiation band.

where t is temperature, q_{gen} is the rate of internal heat generation, W/m^3; k is thermal conductivity, W/mK; α is thermal diffusivity, s/m^2; and τ is time, s.

Thermal conductivity is an intensive property of material which is a proportionality constant relating heat flux by conduction, and the temperature gradient,

$$k = - q'' / (dt/dn) \tag{3-2}$$

where n is the direction of the positive temperature gradient. Heat flux, q", is thermal energy transferred per unit area per unit time (W/m^2, for example). Thermal conductivity values for real materials are determined empirically. Thermal conductivity may be a function of time, temperature, moisture content, direction, or location. However, for most applications in environmental engineering, it is adequate to assume thermal conductivity is a relatively constant scalar. The negative sign is introduced in Equation 3-2 so a positive heat flux corresponds to a negative temperature gradient. Heat always flows "downhill".

In the SI system, units for thermal conductivity are W/mK. To follow convention, temperature units in thermal conductivity, and other parameters to follow, will be expressed in K when a temperature difference is represented and C for actual temperature. Each degree difference in K is equivalent to a degree difference in C.

Appendix Tables A3-1 and A3-2 contain thermal conductivity data for common engineering materials. P385

Thermal diffusivity is a combination of thermal parameters,

$$\alpha = k / \rho c_p, \tag{3-3}$$

傳熱能力 vs. 蓄熱能力

where ρ is mass density and c_p is specific heat. Thermal diffusivity is a measure of how rapidly thermal energy can penetrate a solid material and is important in time-dependent heat conduction problems.

The Laplacian, $\nabla^2 t$, for one-dimensional heat transfer is

$$\nabla^2 t = \frac{1}{n^m}\frac{d}{dn}\left\{n^m \frac{dt}{dn}\right\}, \begin{array}{l} m = 0 \text{ cartesian} \\ m = 1 \text{ cylindrical} \\ m = 2 \text{ spherical} \end{array} \tag{3-4}$$

Many engineering problems can be analyzed using simplified forms of the heat conduction equation. If there is no source term, the Fourier equation is obtained

$$\nabla^2 t = (\alpha)^{-1}\ \delta t / \delta\tau, \tag{3-5}$$

which applies to time-dependent, conduction heat transfer problems.

If conduction is steady-state, but a uniformly distributed heat source is present, the Poisson equation is obtained

$$\nabla^2 t + q_{gen} / k = 0. \qquad (3\text{-}6)$$

This situation can arise, for example, when solar energy is absorbed within the glass or plastic glazing on a greenhouse.

If conduction is steady-state and there are no internal, uniformly distributed heat sources, the Laplace equation results

$$\nabla^2 t = 0. \qquad (3\text{-}7)$$

The Laplace equation applies to most thermal energy exchanges by conduction in agricultural buildings. Although ambient conditions are, in reality, always time-dependent, agricultural buildings are usually sufficiently lightweight in a thermal sense that steady-state (or step-wise steady-state) conditions may be assumed.

One way to estimate whether a building is lightweight in a thermal sense is to consider how rapidly the building might respond to a sudden change of outdoor temperature. Air temperature in a ventilated barn, for example, would be expected to follow outdoor temperature with a lag of less than 1 hr. The same could be expected for temperature changes of other components of the barn such as the walls. Compare this to the period of the diurnal cycle of outdoor temperature which is 24 hrs. The response time of the barn is much less than the period of the outdoor temperature changes, thus, the barn may be considered thermally lightweight and steady-state analysis would be suitable as an approximation of the actual conduction heat transfer process. (Thermal lag is associated with the conduction process; radiation heat transfer is nearly instantaneous, and convective heat transfer occurs within seconds.) As a rough rule, if the maximum air temperature within a building is reached within 2 hrs of the maximum outside temperature, the building may be classed as a lightweight structure.

3-2.2. Temperature Fields. If two boundary conditions for Equation 3-7 are known, the equation can be solved to determine the temperature field. Example 3-1 shows the solution of Equation 3-7 for one-dimensional heat transfer by conduction through a planar solid, a situation resembling heat loss through a homogeneous wall of a building.

Example 3-1

Problem: Determine the temperature field within a homogeneous wall of thickness, L, when the temperature at one boundary is t_1, and the temperature at the other boundary is t_2.

Solution: This example can be considered a case of one-dimensional heat transfer in cartesian coordinates and Equation 3-7 reduces to

$$d^2t / dx^2 = 0, \tag{3-8}$$

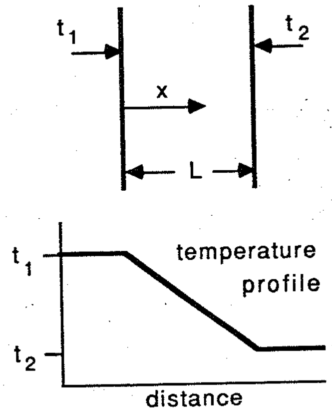
which can be integrated twice to yield

$$t = c_1 x + c_2. \tag{3-9}$$

For $0 < x < L$ the temperature field is

$$t = t_1 + (t_2 - t_1)(x / L). \tag{3-10}$$

Equation 3-10 represents a temperature field linear in distance for the situation of steady-state heat transfer in one dimension in a homogeneous solid with no internal heat sources. Other coordinate systems do not necessarily yield temperature profiles linear in distance.



Conduction heat transfer through cylindrical walls can also be important in building environment analysis and design. For example, a forced hot air furnace may be used to heat a nursery for young animals or a greenhouse. If a duct to carry heated air is round, and is insulated so the wall is thick relative to the duct diameter, conduction heat transfer in cylindrical coordinates must be considered. Example 3-2 demonstrates the solution of such a situation, and a derivation of the expression for temperature in one-dimensional cylindrical coordinates. Heat conduction through insulation on water pipes almost always must be analyzed in cylindrical coordinates.

Example 3-2

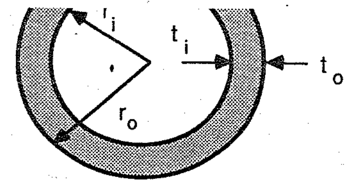
Problem: Determine the temperature field within a homogeneous cylindrical shell of inner radius r_i and outer radius r_o when the temperature of the inner surface is t_i and the temperature of the outer surface is t_o. Assume steady state conditions.

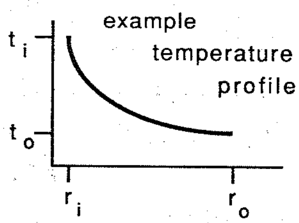
Solution: Equation 3-7 applies again in the form

eq 3-4,

$$\frac{d^2t}{dr^2} + \frac{1}{r}\frac{dt}{dr} = 0, \tag{3-11}$$

m=1





which can be solved readily by substituting $S = dt/dr$. This reduces Equation 3-11 to

$$\frac{dS}{dr} + \frac{S}{r} = 0. \qquad (3\text{-}12)$$

If Equation 3-12 is multiplied by rdr, it becomes

$$r\,dS + S\,dr = 0, \qquad (3\text{-}13)$$

which is, by definition,

$$d(rS) = 0. \qquad (3\text{-}14)$$

Equation 3-14 can be integrated once to

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$$rS = c_1 \qquad (3\text{-}15)$$

and a second time to

$$t = c_1 \ln r + c_2. \qquad (3\text{-}16)$$

Applying the boundary conditions leads to

$$t_i = c_1 \ln r_i + c_2, \text{ and} \qquad (3\text{-}17)$$

$$t_o = c_1 \ln r_o + c_2. \qquad (3\text{-}18)$$

The constants of integration are determined using Equations 3-17 and 3-18, and the temperature field can be expressed as

$$t = \overbrace{\frac{t_o - t_i}{\ln(r_o/r_i)}}^{c_1} \ln r + \overbrace{\frac{t_i \ln r_o - t_o \ln r_i}{\ln(r_o/r_i)}}^{c_2} \qquad (3\text{-}19)$$

Equation 3-19 shows temperature ~~in~~ this steady state case is not linear in distance, but rather shows a logarithmic increase (or decrease, depending on the boundary conditions). Linearity would not be expected intuitively, of course, because the cross-sectional area of heat transfer is a function of radius, thus the temperature gradient must also be a function of radius for steady state heat transfer to occur.

3-2.3. Conduction Heat Transfer. Thermal flux due to conduction heat transfer can be determined by

$$q'' = -k\, dt / dn, \tag{3-20}$$

in a restatement of Equation 3-2 where dt/dn is the temperature gradient and q" is heat flux by conduction. Equation 3-20 expresses the Fourier law of heat conduction. When integrated over the area of heat flow (area normal to the direction of flux) heat flux becomes heat flow (watts, for example).

$J/m^2 \cdot s$ J/s

Under steady-state conditions, Equation 3-20 may be integrated along the path and over the area of heat flow to yield heat flow, q

$$q = kA\Delta t / L;\ \Delta t = t_1 - t_2 \tag{3-21}$$

Equation 3-21 is frequently rearranged and the terms k/L grouped into a single term, U, the unit area thermal conductance,

$$q = UA\Delta t;\ U = k / L. \tag{3-22}$$

Thermal conductance is an extensive property, whereas, thermal conductivity is intensive.

A final rearrangement is frequently made for convenience. The inverse of unit area thermal conductance, termed unit area thermal resistance, R, is used in a restatement of Equation 3-22,

$$q = A\Delta t / R;\ R = L/k. = \frac{1}{U} \tag{3-23}$$

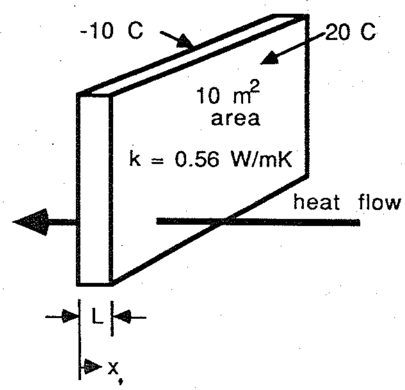
Example 3-3 is a calculation of heat transfer through a single layer wall in cartesian coordinates, Example 3-4 provides a similar calculation for cylindrical coordinates.

see A3-1, A3-2.

Example 3-3

Problem: A wall has been built of concrete and is 200 mm thick with a cross-sectional area of 10 m^2. The temperature of one face is 20 C, the temperature of the other is - 10 C. Assume thermal conductivity of concrete is 0.56 W/mK.

Determine the temperature field and estimate the rate of conduction heat transfer through the wall.



Solution: The temperature field is independent of thermal properties for one-dimensional, steady-state heat transfer by conduction, and properties are uniform and isotropic. Thus Equation 3-10 applies, where $t_1 = -10$ C, $t_2 = 20$ C and $x = 0$ where $t = -10$ C.

$$t = -10 + (20 - (-10))(x/0.2)$$
$$= -10 + 150 \quad \text{(x in m}\cancel{\text{m}}\text{)}$$

$t = t_1 + (t_2 - t_1)\cdot(x/L)$

and by Equation 3-23,

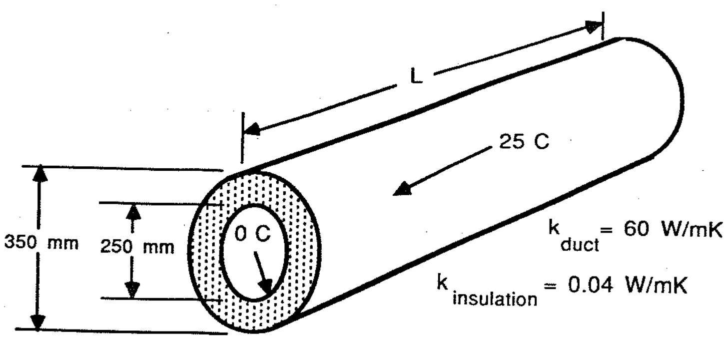
$$R = L/k = 0.2/0.56$$
$$= 0.357\ m^2K/W$$
$$q = A\Delta t/R = (10)(30)/0.357$$
$$= 840\ W.$$

Example 3-4

Problem: A circular sheet metal duct carries refrigerated air to a cold storage room for apples. The duct itself is 250 mm in diameter (outside), and is covered with a 50 mm layer of insulation, making the outer diameter of the insulated duct 350 mm. The duct wall thickness is 1 mm.

The inner surface temperature of the duct is 0 C, and the outer surface temperature of the insulation is 25 C. Thermal conductivity of sheet metal is 60 W/mK, and of the insulation is 0.04 W/mK.

Calculate the rate of conduction heat gain through the insulation per meter length of the duct. Assume steady-state conditions.



Solution: In cylindrical coordinates, thermal resistance to conductive heat transfer is

由 3-23 与 3-19 可导出 →

$$R = \frac{\ln(r_o/r_i)}{2\pi kL} \qquad (3\text{-}24)$$

where L is the length of the cylinder, and r_o and r_i are the outer and inner radii, respectively. It can be a useful exercise to develop this expression using the Fourier law of heat conduction, the definition of heat transfer using the resistance analogy (Equation 3-23), and the equation for the temperature field in cylindrical coordinates (Equation 3-19).

Note that Equation 3-24 presents thermal resistance differently than is the typical case for cartesian coordinates. For the cylinder, resistance is total thermal resistance for the cylindrical wall, not unit area or unit length thermal resistance (unless L = 1). This is an important distinction which will be emphasized later.

This example is to calculate heat gain per meter of duct, thus, L is 1 m in Equation 3-24, k is 0.04 W/mK for the insulation, and r_o and r_i are 175 and 125 mm, respectively. For now, consider just the insulation layer; it is likely to be the limiting resistance along the path of heat transfer. The duct wall itself will have relatively little resistance.

The resistance of one meter length of insulation is

$$R = \ln(175/125)/(2\pi)(0.04)(1.0) = 1.34 \text{ mK/W}$$

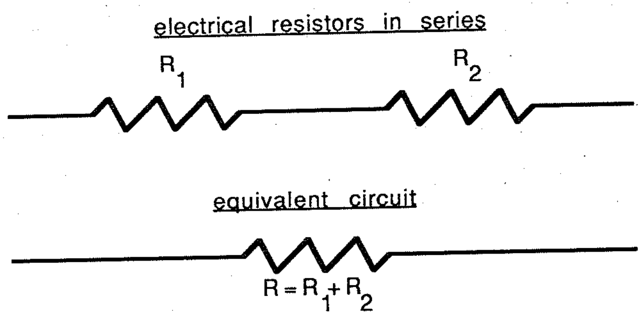
and the heat gain per unit length is

$$q = \Delta t/R = (25 \text{ K})/(1.34 \text{ mK/W}) = 18.7 \text{ W/m}.$$

3-2.4 Resistances in Series. If total resistance, R′, is considered instead of unit area thermal resistance, conduction heat transfer in steady state through more complex geometries resembles electrical current flow through an electrical circuit of resistors, and

current = difference in potential/total resistance or

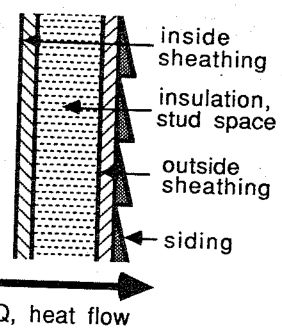
$$q = \Delta t / R'. \tag{3-25}$$



The electrical analogy is useful for realistic conduction heat transfer problems; standard electrical circuit equations can be used to simplify thermal circuits. In an electrical circuit with resistors in series, the total resistance of the circuit equals the sum of the individual resistances,

$$R' = \Sigma R_{individual}. \tag{3-26}$$

A typical wood frame wall in a building has several layers. There is inside sheathing, a wall cavity filled with insulation, exterior sheathing, and siding. It is likely no two layers are made of the same material, and each is a different thickness.



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边板

Conduction heat transfer is through each of the layers in series. This is a <u>series thermal circuit</u> and the total resistance of the wall equals the sum of resistances of the layers. A typical calculation is in Example 3-5.

Example 3-5

<u>Problem:</u> A wall is constructed as shown in Figure 3-3. Calculate the unit area thermal resistance of the wall section, and the heat flux which would result if one side of the wall were held at 20 C and the other at - 5 C.

<u>Solution:</u> This is a series thermal circuit; Equation 3-26 can be used to determine the total unit area thermal resistance of the wall, and Equation 3-25 can then be used to determine thermal flux. The first step is to calculate the unit area thermal resistance of each layer:

$$R_1 = L_1/k_1 = (0.020\text{m})/(0.25\text{W/mK}) = 0.08 \text{ m}^2\text{K/W},$$

$$R_2 = L_2/k_2 = (0.100\text{m})/(0.15\text{W/mK}) = 0.67 \text{ m}^2\text{K/W},$$

$$R_3 = L_3/k_3 = (0.030\text{m})/(0.20\text{W/mK}) = 0.15 \text{ m}^2\text{K/W}.$$

$R = \frac{L}{kA}$

单位 K/W

The total unit area thermal resistance is

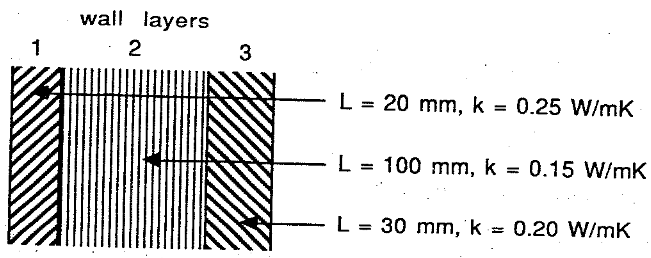
$$R' = 0.08 + 0.67 + 0.15 = 0.90 \text{ m}^2\text{K/W}.$$

$Q = \Delta t / R$

W

The heat flux is

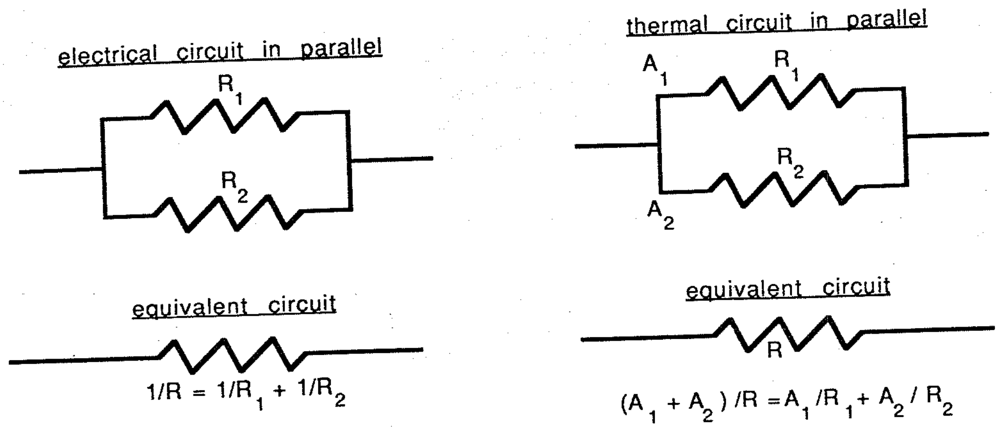
$$q' = \Delta t / R' = (20\text{ C} - (-5\text{ C}))/0.90 \text{ m}^2\text{K/W} = 27.8 \text{ W/m}^2.$$



At this point it would be a useful exercise to return to Example 3-4, calculate the thermal resistance of the sheet metal duct wall, compare that value to the resistance of the insulation and sheet metal in series, and judge whether it was a reasonable assumption to neglect the effect of the duct wall on heat gain into the duct.

3-2.5. Resistances in Parallel. When electrical resistors are in parallel, the total resistance, R´, is found by the inverse rule,

$$1/R' = \Sigma\,(1 / R_{individual}). \qquad (3\text{-}27)$$



A typical building has numerous heat loss paths operating in parallel. Heat transfer through the walls, windows, ceiling, doors, and floor are heat transfer paths in parallel. Each path bridges the same temperature difference – indoor air temperature to outdoor air temperature.

Note: When the electrical analogy is applied to heat transfer paths in parallel, the resistances used in Equation 3-27 must be the total resistances of each path not the unit area resistances. The parallel heat transfer resistance relationship is normally stated as

$$A_{total} / R' = \Sigma\,(A_{individual} / R_{individual}) \qquad (3\text{-}28)$$

where A_{total} is the sum of areas of the heat loss paths,

$$A_{total} = \Sigma\, A_{individual}, \qquad (3\text{-}28a)$$

and $R_{individual}$ is the unit area thermal resistance of each path. In Equation 3-28, R´ is the unit area thermal resistance averaged over all heat transfer paths. Example 3-6 demonstrates an application of parallel heat transfer calculations.

Example 3-6

Problem: A wood-framed wall in a building (see Figure 3-4) is well insulated. However, framing occupies 20% of the wall area, and framing does not have the insulative effect of insulation.

Heat loss through such a wall is a situation of conductive heat transfer paths in parallel. One path of heat loss is through the framed part of the wall, the other path is through the insulated part. For this example, the framed part of the wall has a unit area thermal resistance of 2.3 m^{2}K/W, and the insulated part has a unit area thermal resistance of 4.1 m^{2}K/W.

If the wall is 3 m high and 10 m long, what is the average unit area thermal resistance of the wall, and what will be the heat loss through the wall when it is 20 C indoors and - 5 C outdoors?

Solution: Equation 3-28 is used to calculate the average unit area thermal resistance. There are two paths of heat loss. The total area of heat loss is 30 m^2 (3 m x 10 m). Framing occupies 20% of the wall, a heat loss area of 6 m^2 (0.20% of 30 m^2). The insulated part of the wall is the rest, 24 m^2. Equation 3-28 becomes

$$30\ \mathrm{m^2} / R' = (6\ \mathrm{m^2} / 2.3\ \mathrm{m^2K/W}) + (24\ \mathrm{m^2} / 4.1\ \mathrm{m^2K/W}).$$

The average unit area thermal resistance is

$$R' = 3.55\ \mathrm{m^2K/W}.$$

The heat loss through the wall can be calculated using Equation 3-23,

$$\begin{aligned} q &= (30\ \mathrm{m^2})\,(20\ \mathrm{C} - (-5\ \mathrm{C}))\,/\,3.55\ \mathrm{m^2K/W}, \\ &= 212\ \mathrm{W}. \end{aligned}$$

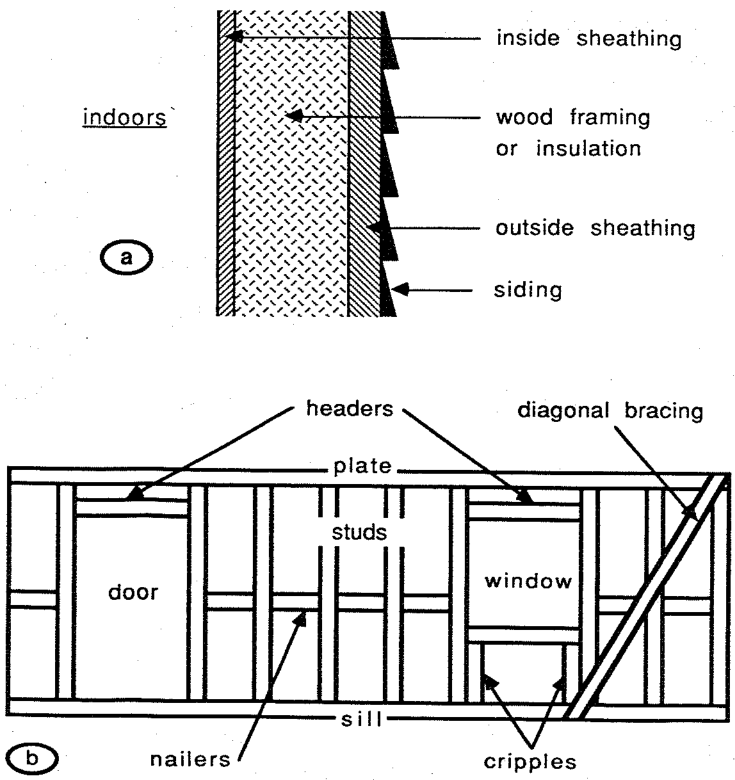


Figure 3-4 Cross-sectio...

Note the influence of a small area of relatively low unit area thermal resistance. The insulated wall section unit area thermal resistance of 4.1 m^2K/W is reduced more than 13% to 3.55 m^2K/W by framing which comprises only 20% of the wall.

Examples 3-5 and 3-6 have demonstrated calculations for heat flow paths in series and parallel. In real situations, both calculations are required. A strategy to approach such problems, in a cartesian coordinate system, is first to examine each series heat flow path, calculate its unit area thermal resistance, and then complete the calculation for all the parallel paths. For example, in a building with walls, ceiling, doors, and windows, the areas of each path and the unit area thermal resistance of each path are calculated first. Then the average unit area thermal resistance and total heat loss are calculated. In cylindrical coordinate systems, total thermal resistance must be used rather than unit area thermal resistances. Modifications of this strategy work, also, but should be attempted only after this straight forward sequence is well understood.

Data to calculate conduction heat transfer exist in two forms. In many engineering applications, thermal conductivity values are used along with Equation 3-21. However, in buildings, many materials are used in standard modules and unit area thermal resistances (R-values) of the modules appear in standard tables. Appendix Table A3-2 contains such a collection of mixed data. Where a material is not used in standard thickness (e.g., concrete) resistance per meter thickness is presented. Where a standard module thickness is typical, the R-value of that module is listed.

The R-value of a desired thickness of material for which an R-value per meter is given can be obtained by scaling the unit thickness R-value. For example, concrete with sand and gravel aggregate, not oven dried, has an R-value of 0.56 m^2K/W per m thickness. A wall 0.2 m thick would, therefore, have an R-value of 0.112 m^2K/W (20% of 0.56 m^2K/W). This calculation, in effect, invokes the concept of resistances in series.

On the other hand, the R-value for Douglas fir plywood, 15.88 mm thick, is given directly as 0.145 m^2K/W. However, if a thickness of material is to be used that is not listed in the table, for example, 10 mm thick Douglas fir plywood, its R-value can be estimated by scaling from given data. The data state 15.88 mm of the plywood has an R-value of 0.145 m^2K/W, thus, 10 mm must have an R-value of 0.091 m^2K/W (= (10 / 15.88) (0. 145m^2K/W)) .

Rounding differences may give slightly different values depending on which database one starts with. If a range of values is given, the midpoint should be used unless manufacturer's data suggest otherwise.

3-3. Convective Heat Transfer

3-3.1. Natural Convection. Convective heat transfer occurs due to a temperature difference between a solid object and its surrounding fluid. In natural convection, fluid moves because of density differences caused by temperature, humidity, or other air constituent gradients. Many engineering studies have been presented which provide empirical correlations between convective heat transfer rates and various thermal and geometric properties of the fluid and solid object.

As an example of natural convection, consider the air duct described in Example 3-4. Cold air within the duct causes the exterior of the duct to be colder than the surrounding room air (if the duct is inside a heated space). Air in the vicinity of the duct cools because of contact with the duct surface, becomes slightly more dense than the surrounding air, and begins to fall. When the cooler air falls away from the duct, it is replaced by room air which is still warm. That air cools, falls, and the cycle is repeated as long as the temperature difference between the air and duct is maintained.

Density differences enhance fluid movement, viscosity retards it. Fluid properties which influence the rate of convective heat transfer are: thermal conductivity, the coefficient of thermal expansion, viscosity, density, and specific heat. The gravitational constant, g, as well as the size and shape of the solid object involved in the heat exchange also influence the rate of convective heat transfer.

In heat transfer, as in fluid mechanics, analyses of physical phenomena are described in terms of dimensionless ratios of the properties and parameters which influence and describe the phenomena. In natural convection, three dimensionless numbers are important.

The first is the Nusselt number, Nu, calculated as

$$Nu = hL/k, \tag{3-29}$$

where L is a characteristic dimension of the solid object involved, k is the thermal conductivity of the fluid, and h is the coefficient of convective heat transfer, also called a film coefficient or a surface coefficient. The characteristic dimension could be the diameter of a horizontal, circular duct, the height of a wall, or the equivalent diameter of a non-circular duct. The Nusselt number can be interpreted as the ratio of the ease with which heat is transferred by convection to the ease with which heat is transferred by conduction, and thus is a measure of the intensity of the convective heat transfer process.

The coefficient, h, is a conductance defined as the flux of heat transferred convectively divided by the temperature difference between the fluid and solid,

$$h = q''/\Delta t, \text{ or } h = (q/A)/\Delta t, \tag{3-30}$$

where q is total heat transferred over an area, A, between a solid object and a fluid due to a temperature difference, Δt.

If the Nusselt number can be determined, convective heat transfer can be quantitatively predicted. 若 Nu 可得, h 可得, q 可求.

The second dimensionless member, the Prandtl number, Pr, expresses the ratio of the diffusion of momentum to the diffusion of heat from a solid surface through a boundary layer into a surrounding fluid. It is calculated as

$$Pr = \mu c_p / k = \frac{\mu}{\rho}\left(\frac{\rho c_p}{k}\right) = \frac{\mu/\rho}{\alpha} \qquad (3\text{-}31)$$

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where, μ is the dynamic viscosity of air, c_p is its specific heat, and k is its thermal conductivity. Example air properties, and the corresponding Prandtl numbers as functions of temperature, are in Table 3-1.

The third dimensionless ratio, important in natural convective heat transfer, is the Grashof number, Gr. This ratio can be interpreted as the ratio of buoyancy forces to viscous drag forces within the fluid,

$$Gr = g\rho^2 \beta L^3 \Delta t / \mu^2, \qquad (3\text{-}32)$$

where β is the coefficient of thermal expansion, g is the gravitational constant, and other terms are as previously defined.

In general, natural convective heat transfer processes have been found to follow the relationship

$$Nu = c(GrPr)^n. \qquad (3\text{-}33)$$

Convective heat transfer mirrors fluid mechanics, where there is a laminar domain, a fully turbulent domain, and between laminarity and turbulence is a domain of mixed laminar and turbulent flow. The coefficient and exponent in Equation 3-33 depend on the fluid flow domain. When flow is laminar, n = 0.33. When flow is turbulent, n = 0.25. When flow is in the transition region between laminar and turbulent, n is between 0.33 and 0.25. In this text only laminar and turbulent flow will be considered, and the transition range will not be covered.

Table 3-1. Properties of dry air at standard atmospheric pressure, 101.325 kPa.

Temperature, K	μ, kg/ms	k, W/mK	Pr	ρ, kg/m^3	c_p, kJ/kgK
200	1.329E-5	0.01809	0.739	1.7684	1.0061
250	1.488E-5	0.02227	0.722	1.4128	1.0053
300	1.983E-5	0.02624	0.708	1.1774	1.0057
350	2.075E-5	0.03003	0.697	0.9980	1.0090
400	2.286E-5	0.03365	0.689	0.8826	1.0140

$\alpha = \frac{\mu/\rho}{Pr}$

(Adapted from Sucec, 1985.)

Air is the fluid of interest in environmental control problems, and the temperature range involved is usually quite narrow. This has permitted simplified forms of Equation 3-33 to be developed for specific applications, forms in which fluid property values are used at standard conditions. More general relationships can be found in the *ASHRAE Handbook of Fundamentals*, for example. Table 3-2 contains equations for natural convection useful in environmental control applications, for dry air at standard atmospheric pressure and 20 C, and have been adapted from the *ASHRAE Handbook of Fundamentals*.

The determination whether flow is laminar or turbulent is based on the GrPr product. Laminar range equations apply for GrPr between 10^4 and 10^8, and the turbulent equations apply for GrPr between 10^8 and 10^{12}. For standard air conditions, GrPr can be approximated by

$$\mathrm{GrPr} = 10^8 \underbrace{L^3 \Delta t}_{1 \sim 10^4}, \tag{3-34}$$

where L is in meters and Δt in K.

Examples 3-7, 3-8, and 3-9 demonstrate uses of convective heat transfer calculations. The examples apply to air ducts and pipes. Convective heat transfer to and from walls of buildings is treated differently, as we will see later in the text.

for dry air at 標準大氣壓 and 20°C

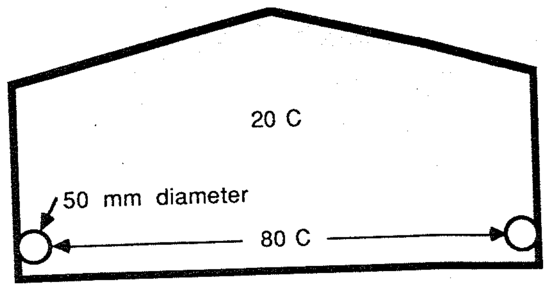
Table 3-2. Natural convention heat transfer coefficients.

垂直板	Vertical plates		
	laminar range	$h = 1.42(\Delta t/L)^{0.25}$	(3-35)
	turbulent range	$h = 1.31(\Delta t)^{0.33}$	(3-36)
水平板 朝上遇冷 or 朝下受熱	Horizontal plates facing upward when cooled or downward when heated (always laminar convective heat transfer)		
		$h = 0.59(\Delta t/L)^{0.25}$	(3-37)
平板朝上遇熱、朝下受冷	Horizontal plates facing upward when heated or downward when cooled		
	laminar range	$h = 1.32(\Delta t/L)^{0.25}$	(3-38)
	turbulent range	$h = 1.52(\Delta t)^{0.33}$	(3-39)
平管	Horizontal cylinders		
	laminar range	$h = 1.32(\Delta t/L)^{0.25}$	(3-40)
	turbulent range	$h = 1.24(\Delta t)^{0.33}$	(3-41)
直管	Vertical cylinders can be treated as vertical plates.		

(Δt in K, L in m, and h in W/m^2K)

Example 3-7

Problem: Consider a horizontal heating pipe for a greenhouse. The pipe carri warm water and releases heat into the greenhouse air by convective he transfer. The outside diameter of the pipe is 50 mm, and its surface temperatu is 80 C. The greenhouse air temperature is 20 C. What will be the surfa convective heat transfer coefficient, and what will be the rate of convective he loss from the pipe?



Solution: The pipe can be considered a horizontal cylinder (no fins on the p were mentioned). However, to know which of the equations in Table 3-2 to u we must determine whether the convection will be laminar or turbulent. Fr Equation 3-34,

$$\begin{aligned} GrPr &= 10^8(0.050\ \text{m})^3(80\ \text{C} - 20\ \text{C}) \\ &= 0.75\text{E}+6 \end{aligned}$$

which is within the laminar range. The convective heat transfer coefficient thus be calculated as (using Equation 3-40)

$$\begin{aligned} h &= 1.32(60\ \text{K} / 0.050\ \text{m})^{0.25} \\ &= 7.8\ \text{W/m}^2\text{K}. \end{aligned}$$

The rate of heat loss can be determined by rearranging Equation 3-3 follows:

$$\begin{aligned} q'' &= h\Delta t \qquad (3 \\ &= (7.8\ \text{W/m}^2\text{K})(60\ \text{K}) = 470\ \text{W/m}^2. \end{aligned}$$

Heat loss from pipes is often expressed per unit length of pipe rather thar area. Each meter of pipe length corresponds to

$$A = 2\pi rL = \pi(0.050\ \text{m})(1.0\ \text{m}) = 0.157\ \text{m}^2$$

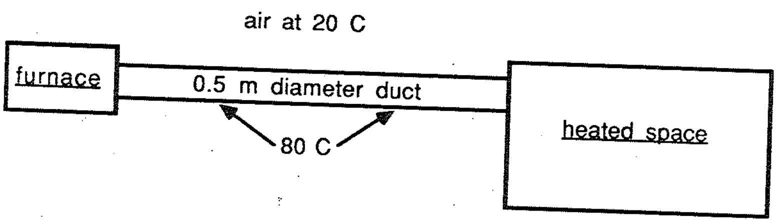
thus,

$$q = (470\ \text{W/m}^2\ \text{K})(0.157\ \text{m}^2/\text{m}) = 74\ \text{W/m}.$$

(This has been a calculation based on basic considerations of convectiv transfer. It should be noted that heat losses from greenhouse heating pip usually calculated based on data provided by manufacturers of heating sy data which have been obtained from experiments, which apply to the specific arrangement of pipes being considered in a design, and which include both convective and radiative heat transfer.)

Example 3-8

Problem: Consider a horizontal sheet metal duct which carries heated air from a furnace to a heated space. The outside diameter of the duct is 0.5 m, and its outside surface temperature is 80 C. The air temperature in the space through which the duct passes is 20 C. What will be the surface convective heat transfer coefficient and the rate of heat loss by convection from the duct?



Solution: This example is similar to Example 3-7, only the dimensions have changed. Again, first check to determine whether laminar or turbulent conditions apply,

$$\begin{aligned} GrPr &= 10^8(0.5\ \text{m})^3(80\ \text{C} - 20\ \text{C}) \\ &= 3.0E{+}9, \end{aligned}$$

which is in the turbulent range. The convective heat transfer coefficient can, thus, be calculated by (using eq 3-41).

$$\begin{aligned} h &= 1.24(60\ \text{K})^{0.33} \\ &= 4.79\ \text{W/m}^2\text{K}. \end{aligned}$$

The rate of convective heat transfer can be calculated as in Equation 3-42,

$$\begin{aligned} q'' &= (4.79\ \text{W/m}^2\text{K})\ (60\ \text{K}) \\ &= 287\ \text{W/m}^2. \end{aligned}$$, $q'' = h \cdot \Delta t$

Each meter of duct has a surface area of

$$A = \pi(0.5\ \text{m})(1.0\ \text{m}) = 1.57\ \text{m}^2,$$

thus, the heat loss by convection is

$$q = (287\ \text{W/m}^2)(1.57\ \text{m}^2/\text{m}) = 451\ \text{W/m}.$$

Compare this example to Example 3-7. Airflow has changed from laminar to turbulent, but the convective heat transfer coefficient has decreased. It is not necessarily true that the average (which is what we have calculated) convective heat transfer coefficient will increase with the onset of turbulence, if the turbulence is caused simply by having a larger solid object. Of course, the much larger size of the air duct leads to a significantly larger rate of heat loss per meter length if not per unit area of duct.

Example 3-9 balance Φ.

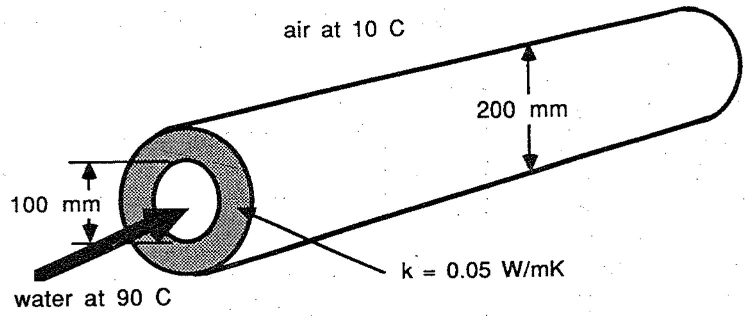
matlab file

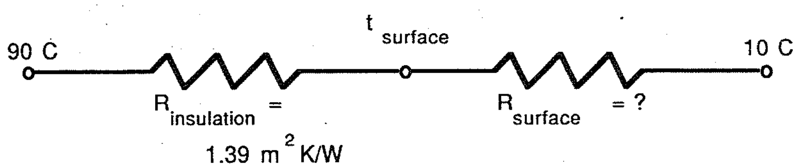
Problem: A hot water pipe is insulated to prevent heat loss from hot water as it flows between the water heater and the point of use.

The pipe's outside diameter is 100 mm, the insulation thickness is 50 mm, the pipe temperature is essentially that of the hot water, 90 C, and air temperature surrounding the pipe is 10 C.

The thermal conductivity of the insulation on the pipe is 0.05 W/mK.

Calculate the heat flux from the surface of the insulation, and the heat loss per meter of pipe.





Solution: Inherent in this problem are assumptions that: (a) the pipe wall does not contribute a significant thermal resistance to the path of heat loss from the water to the ambient air, (b) there is little thermal resistance due to the convective heat transfer process from the water to the inside surface of the pipe, and (c) contact between the pipe and insulation is sufficiently good that there is little contact thermal resistance.

Checking the first assumption is an exercise left for practice. Hint: Use the thermal resistance equations for cylindrical coordinates. Convective resistances from water to a surface are approximately two orders of magnitude less than from air to a surface. Thus, in this problem, the second assumption is reasonable. The third assumption must be based on knowledge of how the insulation will be applied to the pipe; we will assume it will be wrapped tightly and taped.

This problem can be viewed as a thermal circuit with two resistances in series – the insulation, and the convective heat transfer resistance between the surface of the insulation and the air (a surface resistance). The resistance of the insulation can be calculated using Equation 3-24 P. 57

r_o r_i

$$R_{insulation} = (\ln(0.10\ \text{m} / 0.05\ \text{m}))/(2\ \pi)(0.05\ \text{W/mK})(1.0\ \text{m})$$
$$= 2.21\ \text{m K/W}.$$

Recall this thermal resistance is based on a unit length of pipe. Convective thermal resistance is based on unit area. We must decide which basis to use. Either will work, but for this example use unit area. The convective thermal resistance is based on the area of the outside of the insulation layer; the insulation resistance will be based on the same area.

Each meter length of pipe has a surface area of

$$A = \pi(0.20\ \text{m})(1.0\ \text{m}) = 0.628\ \text{m}^2/\text{m}$$

Therefore, the unit area thermal resistance (based on the surface area) is

$$R_{insulation} = (2.21\ \text{m K/W})(0.628\ \text{m}^2/\text{m}) = 1.39\ \text{m}^2\text{K/W}.$$

The thermal circuit is expressed as an electrical analog, and the surface resistance can be determined as the inverse of the convective heat transfer coefficient found using one of the equations in Table 3-2. The question is, which equation? That depends on whether the flow will be laminar or turbulent, and that is a function of the insulation surface temperature. But the insulation surface temperature is a function of the surface resistance. We are in a circle. To approach such a problem, assume a condition, solve the problem using that assumption, and then check the assumption. If the assumption is found to have been incorrect, it is changed and the problem is solved again.

To begin the solution, assume conditions are laminar. An energy balance written for the outside surface of the insulation is

$$h_{surface}(t_{surface} - 10\ \text{C}) = (90\ \text{C} - t_{surface})/ R_{insulation}$$

With the assumption of laminar airflow,

$$h_{surface} = 1.32((t_{surface} - 10\ \text{C})/0.2\ \text{m})^{0.25}$$
$$= 1.97(t_{surface} - 10\ \text{C})^{0.25}.$$

The energy balance can be rewritten as

$$1.97(t_{surface} - 10)^{1.25} = (90 - t_{surface})/1.39.$$

Although this is a single equation in one unknown, it is nonlinear. A simple solution technique is to use trial and error, searching for a value of surface temperature which balances the energy balance. Computers and programmable calculators are well suited to this type of search.

This equation is sufficiently simple that it is most easily solved using a calculator. One approach is to rewrite the energy balance as

$$2.74(t_{surface} - 10)^{1.25} + t_{surface} = 90$$

and search for values of surface temperature until one is found such that the left hand side (LHS) of the equation equals 90. Such a search sequence is

use matlab to solve.

$t_{surface}$	LHS
20 C	68.7
25	105.9
23	90.6
22.9	89.9
22.92	90.0

Considering significant digits, a surface temperature of 23 C is estimated. Now check the assumption of laminar flow.

$$\begin{aligned} GrPr &= 10^8(0.2\ m)^3(23\ C - 10\ C) \\ &= 1.1E+7 \end{aligned}$$

This is within the laminar range; the initial assumption was correct. Several procedures can now be used to calculate heat flux from the pipe. One way is to calculate the surface convective thermal resistance, the total thermal resistance, and the heat flux.

$$\begin{aligned} h_{surface} &= 1.32((23\ C - 10\ C) / 0.2\ m)^{0.25} \\ &= 3.75\ W/m^2K, \end{aligned}$$

and $R_{surface} = 1/3.75\ W/m^2K = 0.267\ m^2K/W$.

The total series resistance is

$$R_{total} = 1.39\ m^2K/W + 0.267\ m^2K/W = 1.66\ m^2K/W,$$

and the heat flux is

$$q'' = (90\ C - 10\ C) / 1.66\ m^2K/W = 48\ W/m^2.$$

Each meter length of pipe has 0.628 m^2 surface area, thus, heat loss per meter is

$$q = (48\ W/m^2)(0.628\ m^2/m) = 30\ W/m.$$

It would be a useful exercise to rework this problem using unit length thermal resistances rather than unit area thermal resistances.

3-3.2. Forced Convection. Forced convective heat transfer is usually greater on a unit area basis than is natural convective heat transfer. Fluid motion caused by a fan or pump is normally more rapid than the rate of motion which can be achieved by thermal buoyancy. As an example of forced convection, air in the heating duct of Example 3-8 moves because of a fan and exchanges thermal energy with the duct wall by forced convective heat transfer. Many forced convection situations are described in heat transfer texts, only a few apply frequently to environmental control calculations in agricultural buildings.

The convective heat transfer equation for forced convection is the same as for natural convection,

$$q'' = h\Delta t,$$

however, the equations which provide values for h differ from those for natural convective heat transfer. The Nusselt number and Reynolds number, Re, are the dimensionless ratios important in forced convective heat transfer. The Reynolds number can be interpreted as the ratio of momentum forces to viscous forces and expresses the level of turbulence,

mass flow rate

$$Re = \rho VL / \mu, \qquad VL / \left(\frac{\mu}{\rho}\right) \tag{3-43}$$

where ρ is mass density, μ is dynamic viscosity, L is a characteristic length, and V is the averaged velocity of fluid flow, $V = \dot{V}/A$.

velocity — Volumetric flow Rate — m^3/s

Environment control applications usually involve air, and forced convective heat transfer applies to airflow inside ducts, for example. The flow is invariably turbulent for realistic situations. For airflow inside a duct at standard atmospheric pressure, the following simplified equation applies,

$$h = c\,G^{0.8} / D^{0.2}. \tag{3-44}$$

In Equation 3-44, c is a coefficient computed from the thermal properties of air and is a function of temperature. Table 3-3 can be used to estimate its value. The parameter G is the mass flow of air in the duct, per unit cross-sectional area of duct, kg/m^2s,

$\rho\dot{V}/A$

$$G = \rho V. \qquad \frac{kg}{m^3}\cdot\frac{m}{s} \tag{3-45}$$

The parameter D is the hydraulic diameter of the duct, m,

$$D = 4(\text{area}) / (\text{perimeter}). \tag{3-46}$$

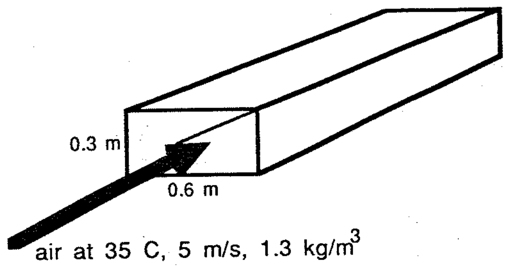
The hydraulic diameter of a round duct equals its physical diameter. Example 3-10 illustrates an application of Equation 3-44, and Example 3-11 is an extended problem which combines forced convective heat transfer and heat conduction.

Generalized correlations, and equations for other fluids and flow situations, can be found, for example, in the *ASHRAE Handbook of Fundamentals*.

Example 3-10

Problem: Air flows through a rectangular heating duct, 0.3 m by 0.6 m. The air velocity is 5 m/s, air density is 1.3 kg/m^3, and air temperature is 35 C.

What is the convective heat transfer coefficient between the heated air and the duct wall?



Solution: To use Equation 3-44, three parameters are needed. The first is c, and its value can be obtained by interpolation from Table 3-3. For a temperature of 35 C, c = 3.23.

The mass flow rate is

$$G = (1.3 \text{ kg/m}^3)(5 \text{ m/s}) = 6.5 \text{ kg/m}^2\text{s},$$

and the hydraulic diameter is

$$D = 4(0.18 \text{ m}^2)/(1.8 \text{ m}) = 0.4 \text{ m}.$$

The convective heat transfer coefficient is

$$h = (3.23)\,(6.5 \text{ kg/m}^2\text{s})^{0.8}/(0.4 \text{ m})^{0.2}$$
$$= 17.3 \text{ W/m}^2\text{K}.$$

Table 3-3. Coefficient c in Equation 3-44 (SI units).

Temperature C	c
-18	3.09
4	3.18
27	3.21
49	3.26
71	3.32
93	3.37

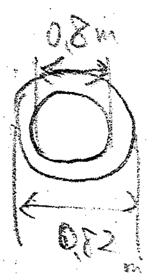
Adapted from the *ASHRAE Handbook of Fundamentals* : for h in W/m^{2}K; G in kg/m^{2}s; D in m. An approximation of the data is the equation c = 3.14783 + 0.00240267t.

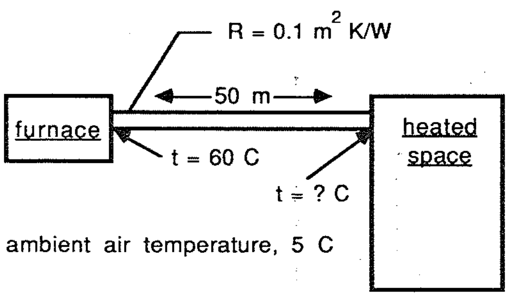
Example 3-11

Problem: Air is heated in a furnace and distributed to a heated space through a round sheet metal duct at a volumetric flow rate of 4 m^3/s. The duct diameter is 0.8 m and the outer surface of the duct is covered with 10 mm of expanded polyurethane having a thermal conductivity of 0.023 W/mK. See P387.

The duct is 50 m long and passes through an unheated space where air temperature is 5 C. The surface resistance outside the duct insulation is 0.1 m^2K/W and includes both convective and radiation heat transfer. Density of the heated air is expected to be 0.9 kg/ m^3.

If air leaves the furnace at 60 C, what will be its temperature at the end of the 50 m long duct? At what rate will heat be lost from the heated air?





Solution: The solution of this problem is not simple, but the example can be solved using only the heat transfer principles covered thus far.

Two steps are required. First, the thermal resistance between air inside the duct and the air outside must be calculated. Then air temperature change along the length of the duct must be determined.

The series thermal circuit from inside the duct to outside is

and $R_{outside}$ is given as 0.1 m^2K/W.

The unit area convective heat transfer coefficient at the inside surface of the duct may be determined using Equation 3-44. At 60 C, c in Equation 3-44 is found in Table 3-3 by interpolation, c = 3.29, and

$$
\begin{aligned}
D &= 0.8 \text{ m},\\
G &= (0.9 \text{ kg/m}^3)\,(4 \text{ m}^3/\text{s})/(\pi)\,(0.4^2)\\
&= 7.16 \text{ kg/m}^2\text{s, and}
\end{aligned}
$$

$$h = (3.29)\,(7.16\ \text{kg/m}^2\text{s})^{0.8}/(0.8\ \text{m})^{0.2}$$
$$= 16.6\ \text{W/m}^2\text{K}.$$

The unit area convective resistance is the inverse,

$$R_{inside} = 1\,/\,16.6\ \text{W/m}^2\text{K}$$
$$= 0.060\ \text{m}^2\text{K/W}.$$

The third resistance in the series thermal circuit is that of the insulation and duct wall. It was stated earlier that a sheet metal wall contributes little to the thermal resistance of an insulated duct, thus, only the insulation will be considered.

The two convective resistances have been calculated on a unit area basis. The conductive resistance must have the same basis to be comparable, but the conductive resistance equation (Equation 3-24) is based on length. A unit area of duct (outside surface area) has a length of

$$L = 1.0/(0.8\pi) = 0.398\ \text{m}.$$ 1 m²

Equation 3-24 is used to calculate the thermal resistance of the wall (insulation) for a duct length of 0.398 m (r_o= 0.41 m, r_i = 0.40 m)

$$R_{wall} = (\ln(0.41\,/\,0.40))/(2\pi)\,(0.023)\,(0.398)$$
$$= 0.429\ \text{m}^2\text{K/W}.$$

(It could be anticipated that a calculation of thermal resistance of the wall in cartesian coordinates might be adequate for this example because the insulation thickness is much less than the radius of the duct. In fact, R = L/k = 0.01 m/ 0.023 W/mK = 0.435 m²K/W is very close.)

A second correction (although small in this example) will be made. The outside surface convective resistance and the wall thermal resistance have been based on the outside surface area. The convective resistance of the inside surface has been based on the inside area. The two areas are not equal. Although the difference in this example is small, it can be significant in other problems.

The correction is 以外径上的 1m² 为基准.

$$R_{inside}(\text{corrected}) = (0.060\ \text{m}^2\text{K/W})(0.82\ \text{m}\,/\,0.80\ \text{m})^2$$
$$= 0.062\ \text{m}^2\text{K/W}.$$

One would expect this increase intuitively. There is slightly less than a unit area of surface inside the duct for each unit area outside. The resistance for an area less than one unit should be greater than the resistance for a unit area, and areas scale by the square of the ratio of their diameters.

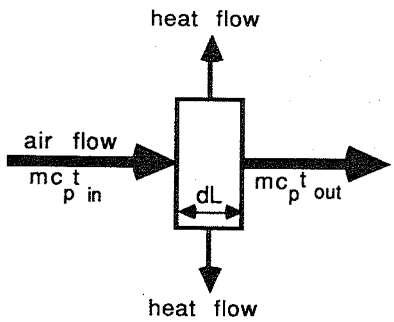
The total resistance of the series thermal circuit is

$R_{total} = 0.062 + 0.429 + 0.10 = 0.591\ m^2K/W$

based on surface area outside the insulation.

The next step, to find the temperature change of the air, requires an application of calculus. The temperature difference between air inside the duct to outside changes continuously along the length of the duct. A simple heat loss equation will not apply (unless the change is very slight, a situation we do not know at this point).

Consider an elemental length of the duct. Heat loss from air as it traverses the elemental length must equal the transfer of heat through the wall to the outside air. This simple energy balance can be written in integral equation form, solved, and used to determine air temperature at any point along the duct.



空气在x方向

Heat transfer through the wall of the elemental length can be written (Equation 3-23) as

$$q = A\Delta t / R;\ A = \pi DdL = 0.82\pi dL = 2.58dL.$$

We have calculated the unit area thermal resistance, $R_{total} = 0.591\ m^2K/W$, thus,

✓ $q = 4.36(t_{air} - 5\ C)dL.$ ← $\frac{2.58\ dL}{0.591}$ · Δt

This thermal exchange must be balanced by heat loss from the mass of air flowing through the element, m,

$$q = -\ mc_p dt_{air};\ m = (0.9\ kg/m^3)\ (4\ m^3/s) = 3.6\ kg/s.$$

The negative sign is introduced so a positive heat loss is associated with a negative temperature change.

If the specific heat of air within the duct is approximated as 1006 J/kgK, heat loss can be written as

✓ $q = -\ 3620dt_{air}.$ $3.6 \cdot 1006 \cdot dT_{air}$

Heat loss must equal heat gain, thus,

$4.36\ (t_{air} - 5)\ dL = -\ 3620 dt_{air}$ $-0.12E\text{-}2\ dL = \frac{1}{t_{air}-5} \cdot dt_{air}$

which can be rearranged and integrated along the 50 m length of the duct in the form

$$\int_{60\,C}^{t_{exit}} \frac{dt_{air}}{(t_{air} - 5)} = -\int_{0\,m}^{50\,m} \overset{0.12E\text{-}2}{\cancel{0.112E\text{-}2}}\ dL$$

$\ln(t_{exit}-5) = \ln(60-5) + -0.06$

and solved as

$t_{exit} - 5 = 55 \cdot \exp(-0.06)$

$$t_{exit} = 5 + 55\ \exp(-\overset{0.06}{\cancel{0.056}}) = \overset{56.79}{\cancel{57.0}}\ C.$$

$t_{exit} = 5 + 55 \cdot \exp(-0.06)$

The energy loss equation can be used again to estimate the rate of heat loss from the air where Δt is now a temperature change not a temperature difference.

$$\begin{aligned} q &= mc_p\Delta t \\ &= (3.6\ kg/s)(1006\ J/kgK)(60\ C - \overset{56.79}{\cancel{57.0}}\ C) \\ &= \overset{11625}{\cancel{10{,}900}}\ W\ (\text{or } \overset{11.6}{\cancel{10.9}}\ kW). \end{aligned}$$

$\frac{129}{0.36 \cdot 1006 \cdot 0.591} = 0.06$

(A natural next question is whether the cost of added insulation would be balanced by the value of heat energy saved.)

In general terms, air temperature in a process such as this example can be calculated from

$$t_{exit} = t_{ambient} + (t_{initial} - t_{ambient})\ \exp(-A/mc_pR)$$ $\exp\left(-\frac{UA}{mC_p}\right)$

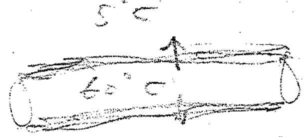
where $t_{ambient}$ is ambient air temperature, $t_{initial}$ is the temperature of air entering the duct, A is the surface area of the duct, m is the mass flow rate of air through the duct, c_p is the specific heat of air, and R is the unit area series thermal resistance of the heat transfer path from inside the duct to outside.

Because of the small temperature change of the warm air as it traverses the duct, we could have avoided the differential equation approach and used a constant temperature difference of (60 C - 5 C = 55K) to calculate heat loss from the air. However, it is useful to see the general approach for use in other applications where temperature changes might be greater.

The simpler approach provides a check on the accuracy of the first approach. The unit area series thermal resistance is 0.591 m^2K/W, and the total area of heat transfer is $\pi(0.82\ m)(50\ m) = 129\ m^2$.

Thus

$$\begin{aligned} q &= A\Delta t / R = (129\ m^2)(55\ K) / 0.591\ m^2K/W \\ &= 12{,}000\ W \end{aligned}$$



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which is somewhat greater than the first estimate because air has not been permitted to cool along the length of the duct.

3-4. Radiation Heat Transfer

3-4.1. General. As has been stated, all objects at temperatures above absolute zero emit thermal radiation. Emission is over a wavelength band, as shown in Figure 3-5. The emissive power shown in Figure 3-5, W/micron, is the thermal energy emitted within each micron waveband. The total emitted thermal energy is the integral or the area under the curve. Approximately a quarter of the emitted thermal energy is at wavelengths shorter than the maximum; three-quarters is at wavelengths above the maximum. As temperature increases, the peak becomes more sharply defined.

The wavelength for peak emission intensity is found using Wien's law,

$$\lambda_{max} = 2898 / T, \tag{3-47}$$

where λ_{max} is wavelength in microns and T is the surface temperature, K, of the emitting object. In Figure 3-5, the curve for a temperature of 300 K peaks at a wavelength of 9.66 microns, for example. Thermal radiation from objects at earth temperature is loosely referred to as having a wavelength of approximately 10 microns. It is often termed "long-wave" or "low-temperature" thermal radiation. The curves in Figure 3-5 show the peak emission of objects at earth temperature is at approximately 10 microns, but significant emission occurs well below and well above this wavelength. Solar radiation peaks at 0.6 microns which represents an effective emitting temperature of 4800 K.

The radiant fluxes emitted from objects at temperatures shown in Figure 3-5 equal the integrals of the curves shown, and can be calculated using the Stephan-Boltzmann relationship,

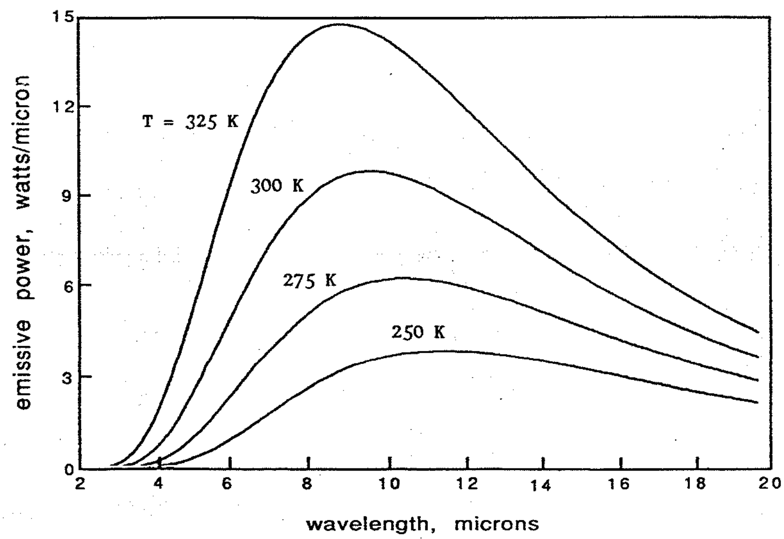

Figure 3-5. Monochromatic emissive power of black bodies at 250, 275, 300, and 325 K.

$q'' = 459$ when $T = 300$

$$q'' = \sigma T^4. \tag{3-48}$$

The constant, σ, is the Stephan-Boltzmann constant, 5.6697E - 8 W/m^2K^4.

The intensity with which earth-temperature thermal radiation is emitted is frequently overlooked. Consider Figure 3-6, in which is graphed thermal flux emitted according to Equation 3-48. Also graphed is the solar constant, which is the intensity of direct-normal solar radiation just outside the earth's atmosphere. The intensity of direct normal radiation on the earth's surface is seldom more than 70% of the solar constant, and then only at high altitudes on very clear, dry days. One-half the solar constant is more typically received on a sunny day, and then only at midday. Figure 3-6 demonstrates that, at temperatures frequently dealt with in building environments (280 to 300 K), the intensity of emitted thermal radiation is approximately one-third the solar constant and compares to solar intensity at sea level on all but the clearest days. So why doesn't everyone freeze to death by loss of thermal radiation? The answer lies in radiation heat transfer calculations, and the exchange of thermal radiation, not just the loss.

3-4.2. Emitted Thermal Radiation. Equation 3-48 applies to a perfect emitter, frequently called a black body. However, no object is a perfect emitter; real substances are characterized by an efficacy of radiant emission, the emittance ε. Radiant energy flux from real objects is calculated from

$$q'' = \varepsilon\sigma T^4. \tag{3-49}$$

Emittance values are normally a function of wavelength. That is, an object will radiate thermal energy with a different efficacy at one temperature than at another. If emittance is not a function of wavelength, the object is termed a gray body. Emittance does not change rapidly as a function of wavelength, thus gray body radiation is assumed to apply in environmental control calculations involving thermal radiation, i.e., calculations over a relatively narrow waveband.

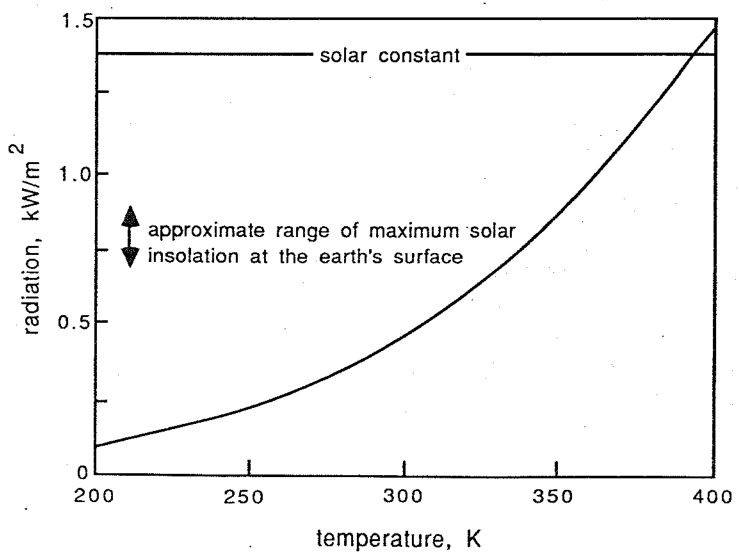

Figure 3-6. Thermal radiation emitted from a black body as a function of its temperature.

It should be emphasized that thermal radiation is a phenomenon determined by the surfaces of objects. Radiation properties of an object are determined by its surface to a depth of only several wavelengths of the radiation involved. As a consequence, thermal radiation properties of a surface can be changed by simply painting over it. Thermal radiation emittance data for common materials can be found in Appendix 3-3. An application of thermal radiation calculations is in Example 3-12.

Example 3-12

Problem: A steam heating pipe in a greenhouse has a surface temperature of 90 C. The surface has been painted with aluminized paint, $\varepsilon = 0.45$. What is the radiant flux leaving the surface, and by how much would the flux change if the pipe were repainted with an oil base or latex paint having an emittance of 0.95?

Solution: The Stephan-Boltzmann equation, 3-49, applies. The pipe surface temperature is 90 C + 273.15 = 363.15 K. The heat flux with the aluminized paint is

$$\begin{aligned} q'' &= (0.45)\,(5.6697E-8)\,(363.15)^4 \\ &= 444\ W/m^2. \end{aligned}$$

When the surface emittance changes to 0.95, the flux increases to

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$$\begin{aligned} q'' &= (0.95)\,(5.6697E-8)\,(363.15)^4 \\ &= 937\ W/m^2, \end{aligned}$$

which is an additional 493 W/m^2.

The heating pipe also loses thermal energy by convection, but this comparison demonstrates the importance of emittance in determining the effectiveness of steam pipe heating systems.

Field experiments have demonstrated a 15 to 20% improvement of steam heating system heat delivery when aluminized paint is covered with ordinary oil or latex paint. Why are so many radiators in homes and steam pipes in greenhouses painted with aluminum paint? There seem to be no reasons other than aesthetics and custom.

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3-4.3. Reflected and Transmitted Thermal Radiation. Reflectance, another thermal radiation property, is defined as the ratio of thermal radiation which irradiates a surface and is reflected to the total irradiation upon the surface (irradiation is defined as the total radiation incident on a surface per unit area and unit time).

A third property, transmittance, is defined as the fraction which through the receiving object. Absorptance, a fourth property, describes the fraction absorbed by the surface (and which is converted to thermal energy). Conservation principles dictate the sum of reflectance, transmittance, and absorptance must be 1.0.

3-4.4. Absorptance and Emittance. Absorptance and emittance are opposites, physically, but they are numerically equal for a given surface and for radiation at the same wavelength. A surface which has an emittance of 0.6 for the wavelength which is characterized by that surface's temperature will also have an absorptance of 0.6 for radiation having the same wavelength. For example, white paint has a solar absorptance which is low – most solar radiation striking white paint is reflected. However, the table in Appendix 3-3 shows white paint has a thermal radiation emittance greater than 0.9, it emits nearly as efficiently as does a black body when it is at earth temperatures. This value of emittance applies to wavelengths near 10 microns. P39

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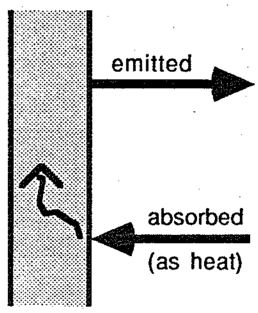


The equality of emittance and absorptance at the same wavelength means white paint also has an absorptance greater than 0.9 for thermal radiation at approximately 10 microns wavelength.

If human eyes were sensitive to wavelengths only between 9 and 10 microns what we now call white paint would appear nearly as black as carbon. In fact, almost all objects would be black – all objects except those with metallic surfaces. This is because, as can be seen in Appendix 3-3, only metals have thermal emittances and thereby thermal absorptances less than approximately 0.9 and a surface with an absorptance that high appears black.

3-4.5. Angle Factors. When two objects exchange diffuse thermal radiation, the net exchange is determined by radiation flux leaving each object as well as the radiation angle factor between the two objects. The angle factor is also termed a shape factor or a configuration factor. The angle factor from one object to another can be interpreted as the fraction of radiation leaving the first object intercepted directly by the second. Another way to interpret the angle factor from, for example, object 1 to object 2, is to imagine how much thermal radiation "universe" of object 1 is occupied by object 2.

Any object exchanging thermal radiation with a second must have a non-zero angle factor with the second object. Angle factor values range from 0.0 (no exchange) to 1.0 (exchange only with the second object). An object can have an angle factor to itself if part of the radiation leaving the surface of that object is intercepted directly by another part of the same object (for example, the inside surface of a hemisphere radiates in part to itself).

Each of any two objects involved in a thermal radiation exchange has an angle factor for the other object, and the two angle factors are not equal. However, a reciprocity relationship can be used to relate them,

$$F_{1\text{-}2}A_1 = F_{2\text{-}1}A_2, \qquad (3\text{-}50)$$

where $F_{1\text{-}2}$ is the angle factor from object 1 to object 2, A_1 is the surface area of object 1, etc.

An angle factor frequently encountered in agricultural engineering environmental analysis applies to thermal radiation exchange between a small object and large surroundings. For example, a cow in a barn and a plant in a greenhouse are small relative to their radiation surroundings. The angle factor from the cow to the walls, ceiling, and floor of the barn is almost 1.0 unless other objects in the barn exchange a significant amount of thermal radiation with the cow. The angle factor from a greenhouse plant to the structural cover of the greenhouse may be approximately 0.5; one-half the radiation "universe" of the plant is the cover (roof and walls). The other half is the greenhouse bench and surrounding plants. By Equation 3-50, the angle factors of the barn to the cow and greenhouse structural cover to the plant are small but not zero.

Note: The sum of angle factors from an object to all other objects (possibly including itself) must equal 1.0. This is an expression of the conservation of thermal radiation. All thermal radiation which leaves an object must be directly intercepted by the other objects with which the first object exchanges thermal radiation.

Extensive tables of angle factors can be found in many heat transfer texts. In Appendix 3-4 are graphs of angle factors for several common thermal radiation exchange situations. Configuration 1 could represent a ceiling and floor exchanging thermal radiation, configuration 2 could represent adjacent walls, a wall and a floor, etc. Configurations 3, 4, and 7 could be assumed to represent a small area exchanging thermal radiation with a ceiling or a specific wall. For example, an animal in a barn could be approximated in shape as a box, with each face of the box exchanging thermal radiation with the surfaces of the barn which it "sees" in a thermal radiation sense, or it could be represented as a sphere. Configurations 5 and 6 could, for example, represent a heating pipe in a greenhouse exchanging thermal radiation with a wall or other large surface. P391

Examples 3-13 and 3-14 demonstrate angle factor calculations.

Example 3-13

Problem: One pig is in a barn. The pig has a surface area of 2 m^2. The barn is 10 m by 20 m with 3 m high walls. The pig exchanges thermal radiation with the inside walls of the barn. Estimate the angle factor from the pig to the barn and the barn back to the pig.

Solution: A pig in a barn qualifies as a small object in a large space. The radiation temperature of the animal's surroundings is assumed to be everywhere uniform, thus, it is not necessary to consider thermal radiation exchange from each side of the animal with the barn, and calculate separate angle factors for each part of the animal's surface to each part of the inside of the barn. We will neglect the angle factor of the pig's surface to itself. For example, while the pig stands, each leg will exchange thermal radiation with the other legs, the under side of the abdomen, etc. However, we will assume for the sake of the example that this is a relatively insignificant part of the total thermal radiation exchange of the pig.

The angle factor from the pig to the barn (F_{1-2}) can be immediately estimated as 1.0. In return, the surface area of the walls, floor, and ceiling of the barn must be calculated and is 580 m^2. By Equation 3-50

$2 \times (10 \times 20 + 10 \times 3 + 20 \times 3) = 580$

$$
\begin{aligned}
F_{2\text{-}1} &= F_{1\text{-}2}A_1 / A_2, \\
&= (1.0)(2\ m^2)/(580\ m^2), \\
&= 0.00345.
\end{aligned}
$$

The other object with which the barn exchanges thermal radiation is itself. Because the sum of angle factors must equal unity,

$$F_{2\text{-}2} = 1.0 - 0.00345 = 0.99655.$$

Example 3-14

Problem: For the barn described in Example 3-13, determine the angle factor for thermal radiation exchange between the ceiling and floor, and between the 3 m by 10 m end wall and an adjacent 3 m by 20 m sidewall. First use the angle factor graphs and then the angle factor equations in Appendix 3-4 (the angle factor from the floor to ceiling is configuration 1, for example).

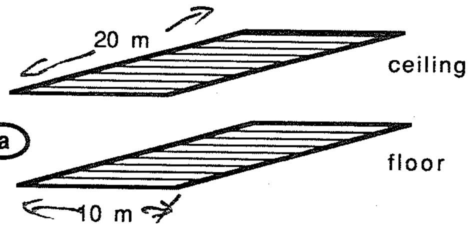
Solution: Appendix 3-4 can be used to determine the angle factor between the ceiling and floor. The ratio a/c is 20/3 or 6.67. The ratio b/c is 10/3 or 3.33. The graph shows the angle factor is approximately 0.65 and is the same from the ceiling to floor as from the floor to ceiling.

The equation corresponding to this angle factor calculation, X = a/c = 6.67 and Y = b/c = 3.33, is

P391

$$F_{1-2}(\pi XY/2) = \ln\left(\frac{(1+X^2)(1+Y^2)}{1+X^2+Y^2}\right)^{1/2} + Y(1+X^2)^{1/2}\tan^{-1}\left(\frac{Y}{(1+X^2)^{1/2}}\right)$$
$$+X(1+Y^2)^{1/2}\tan^{-1}\left(\frac{X}{(1+Y^2)^{1/2}}\right) - Y\tan^{-1}Y - X\tan^{-1}X$$

b=10 a=20 c=3

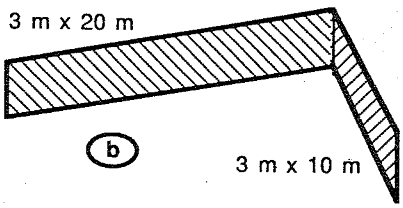


a=3 b=10 c=20 F_{12} = 0.054729

a=3 b=20 c=10 F_{12} = 0.115901

3×20 面 至 A1
c=3
b=20
a=10
F_{12} =

3×10 面 至 A1
c=3
b=10
a=20



The solution for X and Y values as above is $F_{1-2} = 0.66$, approximately the same as was determined from the graph. The symmetry of the problem indicates $F_{2-1} = 0.66$ also. conf. 1.

0.66 + 2× (0.054729 + 0.115901) = 1

The angle factor for thermal radiation exchange between adjacent walls can be found from Appendix 3-4 also. conf. 2. b/c x = a/b = 20/3

If A_1 is the 10 m long wall, the ratio ~~Y/X~~ is 10/3 = 3.33 and the ratio ~~Z/~~X is 20/3 = 6.67. The angle factor from A_1 to A_2 is approximately 0.13. → 0.124

Y = b/c = 20/3

If A_1 is the 20 m long wall, the ratio ~~Y/X~~ is ~~6.67~~ and the ratio ~~Z/~~X is 3.33. The angle factor from A_1 to A_2 is approximately 0.06. x = a/c = 10/3

Note: There are small inaccuracies from reading the graphs, for the two angle factors should relate through reciprocity according to Equation 3-50,

$$F_{1-2} / F_{2-1} = A_2 / A_1.$$

This relationship was approximately confirmed (0.13/0.06 approximately equals 60/30).

As an exercise, use the equation for this geometry as given in Appendix 3-4, simplify it for walls at right angles, and calculate the actual value of the angle factor.

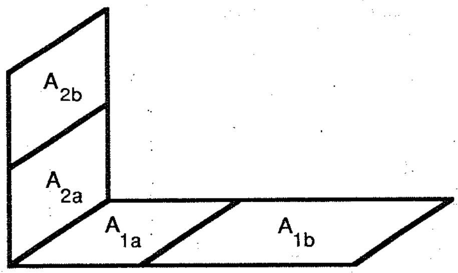
It would also be a useful exercise to determine the angle factors from one wall to all other walls and the ceiling and floor and to check whether they add to unity.

Angle factor algebra can be used to determine angle factors for more complex situations than in Examples 3-13 and 3-14. Calculations are based on conservation of thermal radiation. If diffuse thermal radiation leaves surface 1 and is intercepted by surface 2, and if surface 2 is divided into two areas, 2a and 2b, then

$$A_1F_{1\text{-}2} = A_1F_{1\text{-}2a} + A_1F_{1\text{-}2b}. \tag{3-51}$$

The concept embodied in Equation 3-51, along with inventiveness, permits one to determine angle factors for geometries which are seemingly quite different from those listed in angle factor catalogs.

As a simple example of angle factor algebra, consider the situation as shown where the angle factor is desired between areas A_{1b} and A_{2b}.



Applying Equation 3-51,

$$A_{1b}F_{1b\text{-}2b} = A_1F_{1\text{-}2b} - A_{1a}F_{1a\text{-}2b}, \tag{3-52}$$

where $A_1 = A_{1a} + A_{1b}$. Further application of conservation of thermal radiation leads to

$$A_1F_{1\text{-}2b} = A_1F_{1\text{-}2} - A_1F_{1\text{-}2a}, \text{ and} \tag{3-53}$$

$$A_{1a}F_{1a\text{-}2b} = A_{1a}F_{1a\text{-}2} - A_{1a}F_{1a\text{-}2a}. \tag{3-54}$$

Configuration 2 in Appendix 3-4 can be used to determine all angle factors on the right hand sides of Equations 3-53 and 3-54. The areas will be known, thus the two equations can be solved, and in turn, Equation 3-52 used to calculate $F_{1b\text{-}2b}$.

When infinitesimal areas must be integrated to determine finite area angle factors (see configurations 3-6 in Appendix 3-4), the following relations apply:

$$F_{1-2} = \frac{1}{A_1}\int_{A_1} F_{dA1-A2}\,dA_1, \text{ and} \tag{3-55}$$

$$F_{2-1} = \int_{A_1} dF_{A2-dA1}. \tag{3-56}$$

The angle factor differential in Equation 3-56 is determined using the equation form of the appropriate angle factor in Appendix 3-4.

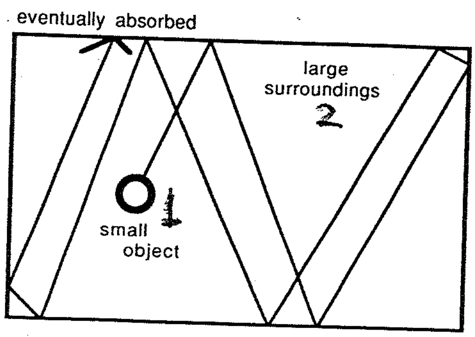
Thermal radiation exchanges are frequently ignored in environmental analyses or are incorporated into empirical coefficients as we will see later. However, when situations arise where thermal radiation is important, and the assumptions built into the empirical coefficients do not apply, a radiation exchange analysis can begin using the information provided here. More detailed thermal radiation exchange information and procedures can be found for example, in Sparrow, and Cess (1978).

3-4.6. Thermal Radiation Exchange. Thermal radiation exchange can be calculated once angle factors are known. A simple situation is the case of a small object in large surroundings. This is simple because once thermal radiation leaves the small object (surface number 1) it will be absorbed by the large surroundings (surface number 2). In other words, surface 2 is thermally black. Even if the absorptance of surface 2 is low, multiple reflections from place to place on surface 2 will eventually absorb all the thermal radiation. Very little will be reflected back to surface 1 because of the very small angle factor from 2 to 1. For this situation, the net exchange of thermal radiation can be calculated,

$q'' = \varepsilon_1 \sigma (T_1^4 - T_2^4)$

$$q_{1-2} = A_1 \varepsilon_1 \sigma (T_1^4 - T_2^4). \tag{3-57}$$

Example 3-15 applies Equation 3-57 to an environmental analysis situation.



Example 3-15

Problem: Consider again Example 3-13. If the pig's skin temperature is 35 C and the average temperature of the calculate the rate of heat loss by radiation from the pig. The emittance of skin can be assumed to be 0.90.

Solution: This is a direct application of Equation 3-57. The skin area of the pig is 2 m^2, and the pig will be termed surface 1. Surface temperatures are:

$$T_1 = 35 + 273.15 = 308.15,$$
$$T_2 = 10 + 273.15 = 283.15.$$

The heat loss is

$$q = (2\ m^2)\ (0.90)\ (5.6697E-8)\ (308.15^4 - 283.15^4)$$
$$= 264\ W.$$

$q = A \cdot \varepsilon \cdot \sigma \cdot T^4 = 2 \cdot 0.9 \cdot 5.6697E{-8} \cdot 308.15^4 \doteq 920.19$

The total thermal radiation leaving the pig is 920W and is found from Equation 3-49. The difference (920 W - 264 W = 656 W) is what returns from the surroundings. This example shows the importance of the thermal radiation environment of an object and even if indoor air temperature is adequate, explains why people feel cold when sitting near a large window in cold weather. The glass is cold and net thermal radiation exchange with it increases heat loss.

More complicated thermal radiation exchange situations require more sophisticated analyses and are most easily implemented on a computer.

To begin, define radiosity, B, as the total radiation which leaves a surface. It is the sum of what is emitted and what is reflected (plus that possibly transmitted)

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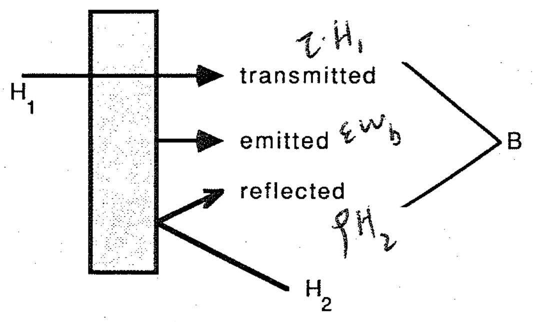
$$B = \varepsilon W_b + \rho H_2\ (+\ \tau H_1) \qquad (3\text{-}58)$$

where W_b is black body emissive power calculated using Equation 3-47, H is irradiation, ρ is reflectance, and τ is transmittance.



When the surface is opaque, $\rho = (1 - \alpha)$ or $(1 - \varepsilon)$, $\tau = 0$, and

$$B = \varepsilon W_b + (1 - \varepsilon)H. \qquad (3\text{-}59)$$

The difference between radiosity and irradiation is the net energy flux lost by an object by thermal radiation (defined as positive if leaving the surface),

$$
\begin{aligned}
q/A &= B - H \\
&= \varepsilon W_b + (\cancel{1} - \varepsilon)H - \cancel{H}.
\end{aligned}
\qquad (3\text{-}60)
$$

From Equation 3-59,

$$H = (B - \varepsilon W_b)/(1 - \varepsilon), \qquad (3\text{-}61)$$

which can be combined with Equation 3-60 to yield

$$q = \varepsilon A(W_b - B)/(1 - \varepsilon), \qquad (3\text{-}62)$$

Equation 3-62 applies to an object exchanging radiation with all objects in its radiation surroundings. Consider the situation with n isothermal surfaces exchanging radiation as would be the case for inner surfaces of walls in a room. For any one surface, i, the irradiation on i is the sum of radiation received from all other surfaces,

$$H_i A_i = \sum_j F_{j\text{-}i} B_j A_j = \sum_j F_{i\text{-}j} B_j A_i \qquad (3\text{-}63)$$

上式去掉 A_i →

$$\text{or } H_i = \sum_j F_{i\text{-}j} B_j. \qquad (3\text{-}64)$$

Summations are taken over all n surfaces participating in the radiation exchange.

This expression for irradiation is substituted into Equation 3-59 to obtain the following set of simultaneous equations relating irradiation on each of the n surfaces.

$$
\begin{aligned}
B_1 &= \varepsilon_1 \sigma T_1^4 + (1 - \varepsilon_1)\sum_j F_{1\text{-}j} B_j, \\
B_2 &= \varepsilon_2 \sigma T_2^4 + (1 - \varepsilon_2)\sum_j F_{2\text{-}j} B_j, \ldots \text{and} \\
B_n &= \varepsilon_n \sigma T_n^4 + (1 - \varepsilon_n)\sum_j F_{n\text{-}j} B_j.
\end{aligned}
\qquad (3\text{-}65)
$$

Equation 3-65 can be rearranged as a matrix equation for convenience,

$$
\begin{bmatrix}
1-(1-\varepsilon_1)F_{1\text{-}1} & -(1-\varepsilon_1)F_{1\text{-}2} & -(1-\varepsilon_1)F_{1\text{-}3}\cdots & -(1-\varepsilon_1)F_{1\text{-}n} \\
-(1-\varepsilon_2)F_{2\text{-}1} & 1-(1-\varepsilon_2)F_{2\text{-}2} & -(1-\varepsilon_2)F_{2\text{-}3}\cdots & -(1-\varepsilon_2)F_{2\text{-}n} \\
 & \vdots & & \\
-(1-\varepsilon_n)F_{n\text{-}1} & -(1-\varepsilon_n)F_{n\text{-}2} & -(1-\varepsilon_n)F_{n\text{-}3}\cdots & 1-(1-\varepsilon_n)F_{n\text{-}n}
\end{bmatrix}
\begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix}
=
\begin{bmatrix} \varepsilon_1 \sigma T_1^4 \\ \varepsilon_2 \sigma T_2^4 \\ \vdots \\ \varepsilon_n \sigma T_n^4 \end{bmatrix}
\qquad (3\text{-}66)
$$

Equation 3-66 is a set of n equations in n unknowns, $B_1, B_2, .. B_n$.

解求 B_i 需知 ε_i, F_{i-j}

Before they can be solved to determine radiosities, all emittances, angle factors, and surface temperatures must be known. Emittances will be known from the choice of materials in the building design. Angle factors can be determined using tables such as in Appendix 3-4 or calculated from the relevant equations. Surface temperatures can be determined using energy balances which will be a later topic in this text. (For now, assume they are known or given.)

T_i

Simple matrix analysis procedures suited for computer implementation can be used to solve Equation 3-66. One candidate solution method is Gaussian elimination, perhaps with pivotal condensation to eliminate the possibility of a pivot near zero, and the resulting inaccuracies.

3-5. Mixed Mode Heat Transfer

Situations frequently arise in environmental analyses where conduction, convective, and radiation heat transfer occur simultaneously. Examples are: (a) heat loss through insulation on a heating duct, where loss on the outside surface is both convective and radiative, and (b) heat gain at a roof's upper surface with loss by conduction to the underside and loss on the upper side by convective and radiative heat transfer. Such problems can be solved in a straightforward manner if a proper energy balance is developed.

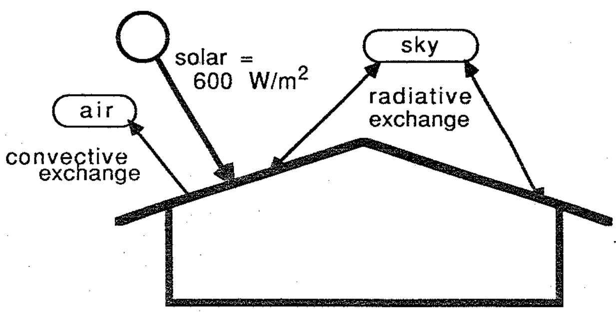
Typically, an energy balance is written for the surface with an unknown temperature and solved for that temperature. Ambient conditions are completely specified. The energy balance is usually nonlinear and may be solved most readily by trial and error. Trial and error or iterative solutions are not elegant, but they work.

Examples 3-16 and 3-17 illustrate mixed process heat transfer problems which can arise in environmental analysis.

Example 3-16

matlab file

Problem: The sun shines on the roof of a barn with an insolation of 600 W/m^2. The absorptance of the roof for solar energy is 0.6.



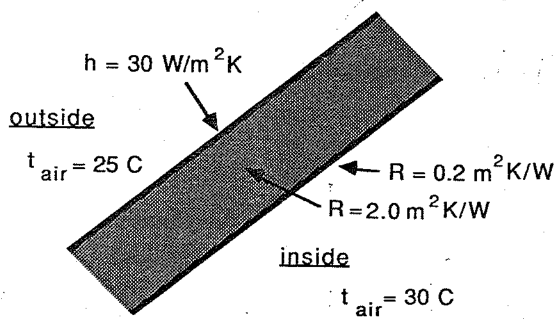
The roof exchanges thermal radiation with the sky. The surface emittance is 0.90, and sky temperature can be approximated by the Swinbank model,

$$T_{sky} = 0.0552 T_{air}^{1.5}, \tag{3-67}$$

where temperatures are expressed in Kelvin degrees.

The roof also exchanges thermal energy by convective heat transfer with the outside air, the convective coefficient is 30 W/m^{2}K. Ambient air temperature is 25 C.

The roof is insulated, having an R-value of 2 m^{2}K/W. Air temperature under the roof is 30 C, and the surface resistance between the lower surface of the roof and air inside the barn is 0.2 m^{2}K/W. This surface resistance includes both convective and radiative resistance.



The temperature of the lower surface of the roof, t_{ls}, is important in determining the comfort of animals within the barn; they exchange thermal radiation with the roof. What will be the temperature of the lower surface of the roof with the given conditions?

Solution: All boundary conditions are known except for sky temperature, and a model is available to calculate that,

$$\begin{aligned} T_{sky} &= 0.0552(25\ C + 273.15)^{1.5}, \\ &= 284.19\ K\ (= 11\ C, \text{ or } 14\ K \text{ below air temperature}). \end{aligned}$$

The temperature of the lower surface of the roof is desired, but before that can be found, the temperature of the upper surface, t_{us}, must be calculated. To determine t_{us} an energy balance on the upper surface of the roof is formed as follows:

$$\begin{aligned} \text{gains} &= \text{losses}, \\ q''_{solar} &= q''_{convective} + q''_{radiative} + q''_{conductive}. \end{aligned}$$

In this energy balance, solar radiation is considered the only gain, other fluxes are losses. The assumptions as to which are gains and which losses are not fixed. The energy balance is correct as long as temperature differences are assigned to agree with the assumed directions of heat transfer.

Absorbed solar flux is calculated as

$$q''_{solar} = (0.60)\ (600\ W/m^2) = 360\ W/m^2.$$

Convective heat loss is

$$q''_{convective} = h\Delta T = 30\ W/m^2K(T_{us} - 298.15\ K).$$

Absolute temperatures will be used in all terms of the energy balance because they are required for radiative heat transfer calculations.

Conductive heat transfer is (R = 2.2 from T_{us} to the inside air)

$$q''_{conductive} = \Delta T/R = (T_{us} - 303.15\ K)/2.2\ m^2K/W.$$

Radiation heat transfer between the barn's roof and the sky can be considered a situation of a relatively small object in large surroundings. Thus thermal radiation loss to the sky can be written

$$\begin{aligned} q''_{radiation} &= \varepsilon_{us}\,\sigma\,(T_{us}^4 - 284.19^4) \\ &= (0.9)(5.6697E\text{-}8)(T_{us}^4 - 65.228E + 8), \\ &= 5.1E\text{-}8T_{us}^4 - 332.7. \end{aligned}$$

All the terms can be substituted into the energy balance and rearranged to solve for T_{us},

$$\begin{aligned} 360\ W/m^2 &= 30(T_{us} - 298.15) \\ &\quad + (T_{us} - 303.15)/2.2 + 5.1(T_{us}/100)^4 - 332.7 \end{aligned}$$

or $\quad 5.1(T_{us}/100)^4 + 30.4545T_{us} = 9775.3.$

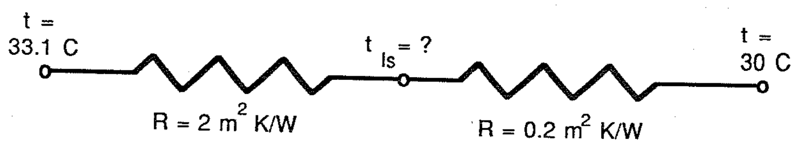
Note: For convenience, T_{us} is divided by 100 to eliminate the need to carry exponents of 10 in the equations. A trial and error solution will be used to determine a value for T_{us} such that the left hand side (LHS) of the last equation equals 9775.3.

T_{us}, K	LHS
310	9911.89
305	9729.96
306	9766.23
306.5	9784.39
306.25	9775.31

T_{us} = 306.2497

We have reached a solution, $t_{us} = 306.25\ K - 273.15 = 33.1\ C.$

Heat transfer through the roof to air within the barn forms a series thermal circuit from which the temperature of the lower surface of the roof, t_{ls}, can be found.



This is a series thermal circuit, thus, temperature differences scale linearly with resistances. The temperature of the lower surface of the roof is

$$
\begin{aligned}
t_{ls} &= 33.1\ \text{C} + (2.0/2.2)(30\ \text{C} - 33.1\ \text{C}) \\
&= 30.3\ \text{C.}
\end{aligned}
$$

The lower surface temperature is only slightly above inside air temperature and solar heating has little effect. This is because the roof is well insulated. As an exercise, do the example again using a roof thermal resistance of 0.3 m^2K/W, an uninsulated case.

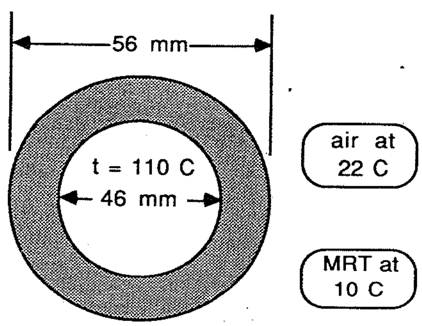
A second useful exercise would be to calculate the magnitude of each term of the energy balance on the upper surface of the roof and determine whether conductive, convective, and radiative heat losses equal the absorbed solar energy when added together, and compare them as to each one's importance as a means of heat loss from the upper surface of the roof.

Example 3-17 balance Φ
balance 1

matlab file

Problem: Greenhouses are frequently heated by steam circulated through iron pipes. The condensing steam contains a great deal of energy which is added from the outer surface of the pipe to the greenhouse by convection to the air and radiation to the interior parts of the greenhouse.

Consider a case where heating pipes with outside diameters of 56 mm are used.



The inside diameter of each pipe is 46 mm. The iron in the pipe has a thermal conductivity of 52 W/mK.

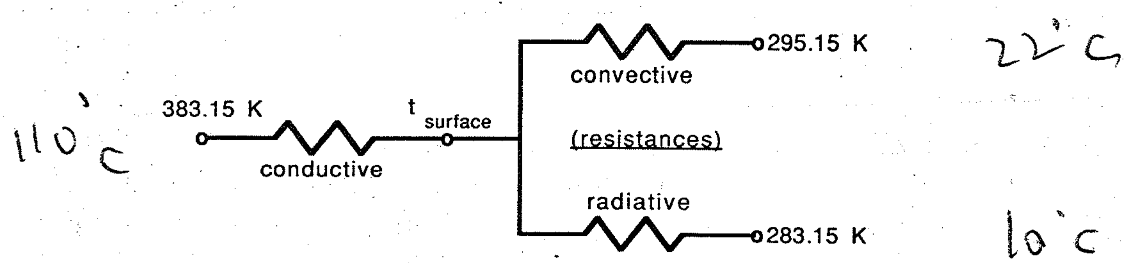
The steam is pressurized and has a condensation temperature of 110° C. Condensation is such a vigorous process that we can assume the inside surface of the pipe equals steam temperature.

The greenhouse air temperature is 22 C and the mean radiant temperature of the surroundings of the heating pipe is 10 C (the pipe is next to a cold outside wall). Mean radiant temperature is the hypothetical temperature which would cause the same net radiative heat transfer with the heating pipe as if the surroundings were all at this mean radiant temperature instead of their many different actual temperatures.

If the heating pipe is painted black and has a surface emittance of $\boxed{0.95}$, what is the net thermal exchange between the pipe and the greenhouse? How much of the heat transfer is convective and how much radiative?

Solution: This is a situation of both series and parallel heat transfer involving conductive, convective, and radiative heat transfer. The convective and radiative transfers from the outer surface are in parallel, and the two together are in series with conductive transfer through the pipe wall. The parallel transfer is somewhat different from what we have seen before. Normally, parallel heat transfer is defined as being between the same temperature difference, but here convective heat transfer is to air at 22 C and radiative heat transfer is to surroundings at 10 C. However, the problem is readily solved using energy balance analysis.

The heat transfer network is as shown (with temperature expressed in Kelvin).



Radiative heat transfer will not be considered a thermal resistance for solution of heat transfer, although in principle it could be. Instead, an energy balance will be developed for the outside surface of the pipe, with the goal of determining the pipe's surface temperature. After that, heat transfer magnitudes will be determined. This strategy is useful to solve many heat transfer problems of this sort, as it was in Example 3-16.

At the pipe's surface,

conductive gain = radiative loss + convective loss.

The conduction heat transfer thermal resistance is found in cylindrical coordinates from

$$\begin{aligned} R_{conductive} &= (\ln(r_o / r_i))/2\pi kL \\ &= (\ln(28/23))/2\pi(52)\ (1) \quad \text{(per unit length)} \\ &= 0.000602\ \text{mK/W}. \end{aligned}$$

Each meter of pipe has 0.1759 m^2 of surface area. For now, calculations will be on a unit area basis. The unit area (based on the outside surface area) resistance of the pipe wall to conduction heat transfer is

$$\begin{aligned} R_{conductive} &= (0.000602\ \text{mK/W})\ (0.1759\ m^2/m) \\ &= 0.000106\ m^2K/W, \end{aligned}$$

and heat gain to the surface by conduction is

$$\begin{aligned} q''_{conductive} &= \Delta T / R \\ &= (383.15\ K - T_{surface}) / 0.000106\ m^2K/W. \\ &= 9441(383.15 - T_{surface}) \end{aligned}$$

The pipe will be assumed to radiate as a small object in large surroundings; the angle factor will be assumed to be 1.0, and Equation 3-51 applies. Loss of heat from the surface by radiation is

$$\begin{aligned} q''_{radiative} &= (0.95)\ (5.6697E - 8)\ (T^4_{surface} - 283.15^4) \\ &= 5.386(T_{surface} / 100)^4 - 346.3 \end{aligned}$$

Convective heat transfer is by natural means, but it is not clear whether it will be by laminar or turbulent transfer. We can be guided in deciding whether to assume laminar or turbulent conditions by the criterion of Equation 3-34. The laminar and turbulent ranges divide at a GrPr value of 10^8, thus, at the dividing value

$$L^3 \Delta T = 1;\ L = 0.056\ m. \tag{3-68}$$

If flow is laminar, the temperature difference between the pipe's surface and the air must be no more than

$$\Delta T = 1 / L^3 = 5694\ K.$$

We can be confident airflow around the pipe, and convective heat transfer, will be laminar.

With laminar heat transfer, the convective coefficient is calculated using Equation 3-40 from Table 3-2, p65.

$$\begin{aligned} h &= 1.32\ ((T_{surface} - 295.15\ K) / 0.056\ m)^{0.25}, \\ &= 2.713\ (T_{surface} - 295.15)^{0.25}. \end{aligned}$$

Convective heat transfer from the surface is

$$q''_{convective} = h\Delta T = 2.713(T_{surface} - 295.15)^{1.25}.$$

The energy balance can now be written (for the surface temperature expressed as T_s)

$$9441(383.15 - T_s) = 5.386(T_s/100)^4 - 346.3 + 2.713(T_s - 295.15)^{1.25}$$

and rearranged as

$$5.386(T_s/100)^4 + 2.713(T_s - 295.15)^{1.25} + 9441T_s - 3{,}617{,}760 = 0$$

which is a single equation with one unknown. As before, a trial and error solution will be used, searching for a value of surface temperature such that the left hand side (LHS) of the rearranged energy balance equals zero. A sequence which leads to the solution is

T_S, K	LHS
382	-9432
383	32
382.99	-63
382.995	-16
382.997	3
382.9967	0.3

In this solution, the value of LHS changes rapidly with small changes of T_S, thus, the value T_S = 382.9967 K (= 109.8467 C) is very close, and convergence need be pursued no further.

In terms of significant digits, as determined by good engineering practice, 382.9967 has too many, but for the purpose of illustration, all decimal places will be carried to check how well the energy balance is satisfied. Note how in this example the surface temperature must be found with considerable care because of the large influence small errors have on the calculated value of conductive heat gain. To obtain accuracy, the constant in the energy balance (3,617,760) must be carried to this number of significant digits because the convergence criterion in the solution of the rearranged energy balance is of the order of unity.

With the surface temperature found, heat transfer magnitudes can be calculated,

$$q''_{conductive} = 9441(383.15 - 382.9967) = 1542 \text{ W/m}^2,$$
$$q''_{radiative} = 5.386(382.9967 / 100)^4 - 346.3 = 813 \text{ W/m}^2,$$

and

$$q''_{convective} = 2.713(382.9967 - 295.15)^{1.25} = 730 \text{ W/m}^2.$$

The energy balance is essentially satisfied. Conductive gain to the surface is

1542 W/m^2, and the sum of losses is 813 + 730 = 1543 W/m^2. This magnitude of error is to be expected, for the conduction heat transfer term is extremely sensitive to small rounding errors. A check to be sure the balance is satisfied is important to prevent inadvertent errors.

Heating per unit length of pipe is frequently desired; such values can be obtained from the above by multiplying each flux by the surface area per meter of pipe, 0.1759 m^2/m. → 271 W/m

The temperature difference between the pipe surface and ambient air is much less than that required for turbulent convective heat transfer; that assumption was correct.

Note the loss of heat from the pipe is balanced between convective and radiative. The large radiative component is important in determining the environment of plants near the pipe. They will be subjected to a significantly warmer environment than will those plants shaded from thermal radiation and may exhibit different growth and timing.

The large radiative term is also significant in leading to a means to save heating energy. If the heating pipe is along the outside foundation wall, a reflective (foil-faced) insulation placed on the inside surface of the wall will reflect much of the radiant energy back into the greenhouse instead of allowing the wall to absorb and lose it to the outdoors.

It would be a useful exercise to analyze the heating situation again using a different value of surface emittance for the pipe (0.40 to represent aluminum paint, for example). Would you expect the convective component to change significantly?

A second useful exercise would be to solve the example using the assumption that the pipe wall's thermal resistance is insignificant and the pipe's outside surface temperature equals the steam temperature. Would you expect much difference in the calculated radiative and convective heat transfer rates?

3-6. Program BALANCE

Repeated solutions of energy balances, such as in Examples 3-16 and 3-17, are tedious. Program BALANCE, which is an executable file containing built-in instructions for its use, provides a quicker means to solve the generalized equation contained in the two examples:

$$\underbrace{A1(T/100)^4}_{\text{radiation}} + \underbrace{A2(T - A3)^{A4}}_{\text{convection}} + \underbrace{A5T + A6}_{\text{conduction}} = 0. \qquad (3\text{-}69)$$

For example, in Example 3-17,
A1 = 5.386,
A2 = 2.713,

A3 = 295.15,
A4 = 1.25,
A5 = 9441, and
A6 =-3,617,760.

In Example 3-16,
A1 = 5.1,
A2 = 0.0,
A3 = 0.0,
A4 = 0.0,
A5 = 30.4545, and
A6 = -9775.3.

Example 3-18 balance

matlab file

Problem: Repeat Example 3-17 and develop a graph to show the effect of the thermal conductivity of the pipe wall on the surface temperature of the pipe.

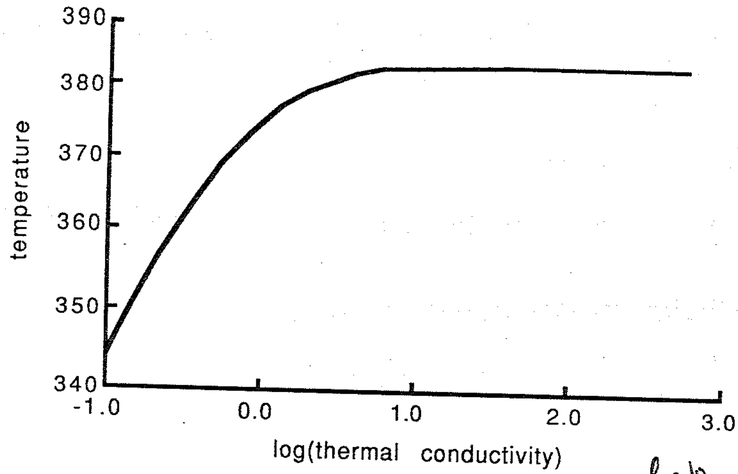
Solution: The thermal conductivity of the pipe wall influences terms A5 and A6 of Equation 3-69. This can be seen by returning to Example 3-17 and reviewing the derivation of the energy balance. For the same mean radiant temperature of the surroundings,

$$A5 = 1/((0.1759\ m^2/m)(\ln(r_o/r_i))/2\pi k) = 181.56k, \text{ and}$$
$$A6 = -346.3 - (383.15)\left(\frac{1}{(0.1759\ m^2/m)(\ln(r_o/r_i)/2\pi k)}\right),$$
$$= -346.3 - 69{,}566k.$$

The thermal conductivity used in Example 3-17 was 52 W/mK. From Appendix 3-1, thermal conductivities span the range from approximately 400 down to approximately 0.05 W/mK. The following values of conductivity lead to the A5 and A6 values shown, and BALANCE provides the surface temperature values.

k, W/mK	A5	A6	surface temperature, K
0.1	18.16	-7,302	342.6
0.5	90.78	-35,129	369.4
5.0	907.8	-348,176	381.5
50	9,078	-3,478,646	383.0
500	90,779	-34,783,346	383.1
0.05	9.078	-3,825	328.41
0.01	1.816	-1,042	303.16

The data, when graphed, provide the following on a semilogarithmic scale. It is obvious that a very low value of thermal conductivity of the pipe wall would be necessary to affect the pipe's surface temperature significantly, a value much lower than typifies metals.



log10.

3-7. Combined Convective and Radiation Surface Coefficients

Calculations for radiation heat transfer differ markedly in form from calculations for convective and conductive heat transfer. A desire to attain greater similarity among the three has led to the concept of a radiation surface coefficient of heat transfer. Although radiation heat transfer is properly calculated as described in previous sections, one can, in concept, propose to estimate radiation heat transfer as

$$q'' = h_r A(T_s - T_a), \tag{3-70}$$

where T_s and T_a are the temperatures of the surface under consideration and the ambient air, respectively. The coefficient, h_r, is defined as the radiation surface coefficient, and is used to calculate the total convective and radiative heat loss from the surface by

$$q'' = \frac{q}{A} = \quad q'' = (h_r + h_c)(T_s - T_a). \tag{3-71}$$

In concept, this is correct for the form of h_r has not been restricted. When heat loss from a surface is expressed in this form, analysis is made easier because radiation heat transfer is linearized and problems of solving nonlinear equations (e.g., Examples 3-16 and 3-17) are avoided. Prior to accessibility to programmable calculators and small computers, avoiding nonlinear problems was a useful goal to achieve.

Linearizing radiation heat transfer can be visualized as an application of a Taylor's series expansion of the Stephan-Boltzmann expression for thermal radiation emission. If the temperature difference between two objects involved in a thermal radiation exchange is small, the expansion may be usefully truncated after the first term leaving an approximating expression for the net exchange between two objects (assume for now only two objects are involved in the exchange, such as a small object within a large enclosure, or one wall of a room exchanging thermal radiation with the other surfaces of the enclosure).

Radiation surface coefficients are obtained from knowledge of actual thermal radiation exchange. For example, consider the previously considered case of a small object in a large room,

$$q'' = \varepsilon_1 \sigma (T_1^4 - T_2^4),$$

where surface 1 is the small object in the large room, surface 2.

Thus, the radiation surface coefficient defined in Equation 3-70 for this situation is

$$h_r = \varepsilon_1 \sigma (T_1^4 - T_2^4)/(T_1 - T_a). \quad (3\text{-}72)$$

Unfortunately, this linearization still involves a nonlinear equation to determine h_r, and h_r is a function of temperatures of the surfaces involved in the heat exchange. If T_2 does not equal T_a, there is little advantage in using a radiation surface coefficient. However, if T_2 and T_a are equal, $A_l << A_2$, and $(T_l - T_a) << T_l$, the following approximation may be made:

$$h_r = 4\varepsilon_1 \sigma T_{ave}^3, \quad (3\text{-}73)$$

where $T_{ave} = 1/2(T_l + T_a)$.

It is frequently assumed that h_r is a constant which can be determined, but in strict terms this is not true. Since nonlinear equations in the forms we have seen can be solved using computers, the need to linearize heat transfer equations is less strong and radiation heat transfer equations can be used directly to avoid the approximation of the radiation surface coefficient.

One situation is still universally used wherein radiation heat transfer is treated using a surface coefficient, and convective and radiative heat transfer are combined as in Equation 3-71. This involves determining the surface coefficients for heat transfers through walls, etc., of buildings. For this situation, convective heat transfer equations such as in Section 3-3 and radiative heat transfer equations such as in Section 3-4 are not used. Instead, surface coefficients which have been determined empirically to represent average conditions for buildings are accepted, radiation is obscured in the process, and heat gain or loss from the surface of the wall, etc., is determined using Equation 3-71 with the empirical coefficient used in place of $(h_r + h_c)$. Appendix 3-5 contains surface coefficient data as presented, for example, in the *ASHRAE Handbook of Fundamentals* (1989).

These combined coefficients will be used repeatedly throughout this text to calculate building heat transfer gains and losses. It is useful at this point, however, to highlight an important assumption inherent in the data of Appendix 3-5. The data apply in a strict sense only when the air temperature involved in

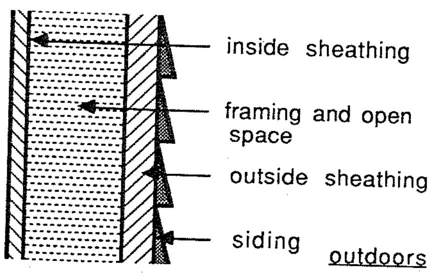
P 399

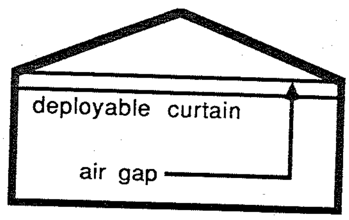
the heat exchange (e.g., the air inside a barn exchanging heat with the inside surface of an outside wall) is the same as the mean radiation temperature of the surroundings of the surface in question.

This is obviously a crude assumption for many applications to agricultural buildings where the temperatures of walls, for example, can differ significantly from air temperature. This would be an especially rough approximation for greenhouses where glass temperature is cold during winter. The data was developed for commercial and industrial buildings with heavy and insulated walls, and in those applications the assumption is more suitable. However, this difference has been frequently overlooked in heat loss calculations for poorly insulated walls, and the errors introduced by the assumption have been accepted.

3-8. Thermal Resistance of Plane Airspaces

A situation which can arise in environmental analysis, and which involves both convective and radiative heat exchange, is the transfer of heat across plane airspaces. One example is loss of heat through an uninsulated wall where the wall cavity is hollow. Another is loss of heat through a night curtain in a greenhouse, where the curtain is composed of at least two layers of fabric or sheet plastic and separated by airspaces.





Values for the thermal resistance of such airspaces have been determined experimentally and are published, for example, in the *ASHRAE Handbook of Fundamentals* (1989). Appendix 3-6 gives thermal resistances of plane airspaces.

P398

The thermal resistance depends on several factors. The mean temperature and the thickness of the airspace, the temperature difference from one side to the other, and the direction of heat transfer are four factors which influence convective heat transfer. Radiative heat transfer is affected by the temperatures of the two surfaces which form the airspace and the emittances of the two surfaces.

It is unlikely both surfaces bounding the airspace will be thermally black, thus an effective emittance, E, of the cavity must be determined.

$$E = (\varepsilon_1^{-1} + \varepsilon_2^{-1} - 1)^{-1} \qquad (3\text{-}74)$$

When the emittances of the two sides of the space are estimated, the cavity emittance is calculated, and the temperatures are estimated, the effective resistance can be calculated for a given direction of heat transfer.

Example 3-19

Problem: Estimate the R-value of just the cavity in a hollow wall framed with lumber which leaves approximately a 90 mm airspace. Consider a situation of winter heat loss from the building where the air temperature inside the building is approximately 20 C and it is - 10 C outdoors.

Solution: Several assumptions are required to obtain a solution. First, it will be assumed the mean airspace temperature is the average of inside and outside temperatures. If we knew more about the wall construction we might be able to obtain a better estimate of the mean temperature, but for now we will use a simple average. We will also assume the temperature difference across the cavity is 10 K. This, again, will depend on the wall construction and how much insulation value is in the inside and outside sheathing, and outside siding, compared to the R-value of the cavity. Finally, assume the materials to sheath the wall are thermally black, their effective emittances for thermal radiation exchange are 0.9.

By Equation 3-74, the effective emittance of the airspace is

$$E = [(1/0.9) + (1/0.9) - 1]^{-1} = 0.82.$$

If this is a building wall, the orientation of the airspace is vertical and heat transfer is horizontal. The tabulated data in Appendix 3-6, for a mean airspace temperature of 10 C, an airspace thickness of 88.9 mm, an effective cavity emittance of 0.82, and a temperature difference of 16.7 K shows a thermal resistance of 0.16 m^2K/W. When the temperature difference is 5.6 K, the resistance is 0.18 m^2K/W. Our assumption of a 10 K temperature difference places us approximately in the middle, thus, we can estimate the actual thermal resistance will be 0.17 m^2K/W.

3-9. Thermal Radiation Exchange with Gases

To this point all thermal radiation exchange has been assumed to occur among solid objects. Gases also emit and absorb thermal energy. The model for sky temperature in Example 3-16, Equation 3-67, is actually a model for the mean radiation temperature of the lower part of the atmosphere. Unless the sky is very clear and dry, thermal radiation exchange with the sky is exchanged only with the lowest few hundred meters of air.

The effective emittance of air depends on both humidity and the carbon dioxide content of the air, both of which can be high in agricultural buildings. The following data are emittance values for carbon dioxide and water vapor at 24 C:

Path length, m	Carbon Dioxide, ppm			Relative Humidity, %		
	1,000	3,000	10,000	10	50	100
3	0.03	0.06	0.09	0.06	0.17	0.22
33	0.09	0.12	0.16	0.22	0.39	0.47

In a barn during times of minimum ventilation, for example, the carbon dioxide level may be greater than 3000 ppm and the relative humidity may be higher than 70%. For an animal in the center of a barn, the effective path length of air surrounding it may be greater than 3 m, thus, thermal radiation exchange with the air may be significant. As a rough rule, the effective path length for air radiating to the walls in an enclosure equals four times the mean hydraulic radius of the enclosure. However, in environment control analyses this factor is usually ignored. With the advent of computers and their processing power, it is more feasible to include radiation exchange calculations in determining the environment provided to animals and plants. However, radiation exchange calculations which involve gases are quite complicated, and thermal radiation exchange with the air is neglected. It is not clear, however, this is a reasonable assumption. Sparrow and Cess (1978), for example, develop analyses for radiation exchange between solid objects and gases, methods which can be useful in exploring the thermal radiation interaction of solid surfaces and air.

SYMBOLS

A	area, m^2
B	radiosity, W/m^2
c	coefficient in Equations 3-33 and 3-44
c_1, c_2	integration constants
c_p	specific heat, kJ/kgK
D	hydraulic diameter, m
E	effective emittance
F	radiation angle factor
g	gravitational constant, m/s^2
G	unit area mass flow rate, kg/m^2s
Gr	Grashof number, see Equation 3-32
h	coefficient of convective heat transfer, W/m^2K
h_r	radiation surface coefficient of heat transfer, W/m^2K
H	irradiation, W/m^2
k	thermal conductivity, W/mK
L	thickness or length, m
n	generalized spacial variable, m
n	exponent in Equation 3-33
Nu	Nusselt number, see Equation 3-29
Pr	Prandtl number, see Equation 3-31

q	heat transferred, W
q''	heat flux, W/m^2
q_{gen}	internal heat generation, W/m^3
r	radial dimension, m
R	unit area thermal resistance, m^{2}K/W
Re	Reynolds number, see Equation 3-43
t	temperature, C
T	absolute temperature, K
U	unit area thermal conductance, W/m^{2}K
V	velocity, m/s
$\dot{V}$	volumetric flow rate, m^3/s
W	thermal radiation emissive power, W/m^2
x	cartesian dimension, m
α	thermal diffusivity, s/m^2
β	coefficient of thermal expansion, K^{-1}
ε	thermal radiation emittance
λ	wavelength, microns
μ	dynamic viscosity, kg/ms
ρ	density, kg/m^3
ρ	reflectance
σ	Stephan-Boltzmann constant, W/m^{2}K^4
τ	time, s
τ	transmittance

EXERCISES

1. Consider a wall made of three layers. The outer layer is four inches of concrete, the inner layer is the same, and the center layer is made of two inches of expanded polystyrene board stock (molded beads, approximately 20 kg/m^3). If the temperatures of the inner and outer surfaces of the wall are 25 and 11 C, respectively, what temperature exists at the interface between the inside and center layers and the center and outside layers? What is the flux of heat through the wall?

2. A hot water heating system is used to heat a calf nursery. Heated water at 70 C flows from the boiler to the nursery through a pipe with an outside diameter of 60 mm. Estimate the convective heat transfer coefficient (based on unit area, and then on unit length) between the pipe surface and the surrounding, still air when the air is at 20 C.

3. Atmospheric air at 60 C enters a sheet metal duct with a cross-section of 0.3 m by 0.5 m. The duct wall is at a temperature of 10 C. During operation, the airflow rate varies between 0.1 and 0.3 m^3/s. Develop a graph of the convective heat transfer coefficient between the air and the duct wall for the range of expected airflow rates.

4. Consider the glass glazing on a greenhouse. The greenhouse is single-

glazed and glass thickness is 8.5 mm. Temperatures of the inside and outside surfaces of the glass are -5 C and -6 C, respectively. Solar insolation with an intensity of 300 W/m^2 strikes the glass (direct normal intensity). Eight percent of the insolation (or 24 W/m^2) is absorbed within the glass. Determine the rate of heat loss from inside the greenhouse to the glazing (W/m^2) when the sun shines, and compare that rate to the rate of loss if there were no absorption of solar energy within the glass.

5. A significant problem related to convective heat transfer in environment control is heat gain or loss from air ducts carrying cold or warm air to conditioned spaces. It is often of interest to determine whether the heat gain or loss is sufficient to warrant concern or a sufficient reason to add insulation.

A refrigeration system provides cooled air for an onion storage room at a food processing plant. For reasons no one can explain, the refrigeration system is located at the opposite end of the building from the storage room – a distance of 50 m. The duct passes through the building on the way to the storage room, and air in the building (surrounding the duct) is at 20 C. The cooled air should arrive at the storage room at 1 C. The question is, what should be the temperature of the air when it leaves the refrigerator?

The problem is one of forced and natural convection – forced inside the duct and natural outside. Radiation and conduction complicate the problem, but for now make the following assumptions:

(a) the thermal resistance and thickness of the duct wall are negligible (it is made of sheet metal and has no insulation),

(b) the combined natural convection and radiation surface coefficient on the outside surface of the duct is 11 W/m^2K,

(c) although the duct will be cold, there will be no condensation on its outside surface. This is obviously not realistic, for there likely will be condensation and associated additional heat exchange, but for now ignore this complicating factor, and

(d) assume the duct is round with a diameter of 0.5 m and airflow at a rate of 0.8 m^3/s. The density of the air inside the duct is approximately 1.3 kg/m^3.

Do the following:

(a) Determine the temperature at which air should be discharged from the refrigeration system to provide 1 C air at the storage room.

(b) Determine the actual natural convection coefficient for heat transfer to the duct from the surrounding still air.

6. A small temperature sensor is used to measure air temperature in an air duct in which hot air flows. The actual air temperature is 95 C, and the actual duct wall temperature is 45 C. The convective coefficient between the air and sensor is estimated to be 150 W/m^{2}K. The surface emittance of the sensor is 0.95.

   The sensor can measure only its own temperature, and the goal of using a sensor is for its temperature to equal the temperature of the medium being sensed. What will be the temperature indicated by the temperature sensor — the apparent air temperature?

7. In a food processing plant, cold water at 3 C is pumped through a 20 mm (outside diameter) galvanized steel pipe. The pipe passes through a room where the air temperature can be as high as 30 C and the relative humidity can be as high as 90%. To prevent condensation on the pipe surface, it is insulated with a sleeve of foam rubber (thermal conductivity of 0.03 W/mK). What thickness of insulation is needed to prevent condensation? (Begin by assuming laminar airflow around the pipe.)

8. Consider a 300 mm diameter, round sheet metal duct insulated on the outside with 50 mm of cellular polyurethane (R-11 exp.). Air moves through the duct at a velocity of 7 m/s, and is at 50 C and 30% relative humidity. The heat transfer coefficient (convective plus radiative) at the outer surface of the insulation is 20 W/m^{2}K; ambient air temperature is 10 C. Calculate the outside surface temperature of the insulation.

√ 9. A single layer night curtain is being used in a large greenhouse. The material of the curtain has no significant thermal resistance in itself; all the resistance is in the convective and radiative properties of the two surfaces. Air temperatures below and above the curtain are 17 C and 8 C, respectively. Radiation temperatures of the greenhouse above and below the curtain are -5 C and 15 C. Emittances of the two sides of the curtain material are 0.20 and 0.90 (one side is aluminum foil, the other is cloth). The greenhouse above and below the curtain can be assumed thermally black ($\varepsilon = 1.0$). For maximum thermal benefit, should the foil side of the curtain be on the upper or lower side of the curtain?

10. Air T below and Above the curtains are 17 C and 40 C. Radiation T above and below are 50 and 15

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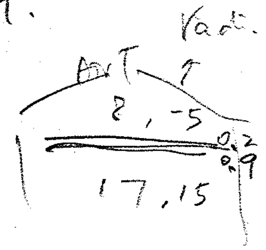

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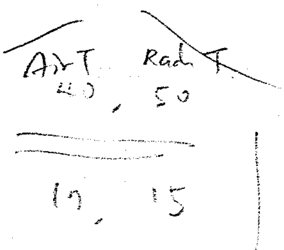

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