# Chap. 5. Steady-State Energy and mass Balance
# 能量與質量守衡之穩態分析

## 5-1. Introduction

建築物環控分析主要涉及三項重要觀念：

1. 控制容積 Control Volumes,
2. 能量守恆 Conservation of Energy,
3. 質量守恆 Conservation of Mass.

Processes in thermodynamics, fluid mechanics, and heat transfer can be viewed in two ways:

1. To examine a process with a focus on its internal features.
2. To enclose the process with an imaginary boundary and examine only what passes across the boundary. (Control Volume approach, black box approach)

顯熱守恆 (Sensible energy balance) 分析主要構成項：(圖 5-1)

| | | | |
|---|---|---|---|
| $q_s$ | 空間中由動物體獲得的顯熱 | $q_w$ | 穿透結構體(牆、屋頂、窗、門、地板、天花板等)所傳入或傳出的顯熱 |
| $q_m$ | 空間中由機械元件獲得的顯熱 | $q_f$ | 穿透地板(主要發生在周邊)所傳入或傳出的顯熱 |
| $q_{so}$ | 空間中由太陽獲得的顯熱 | $q_e$ | 空間中顯熱轉換為潛熱的速率 |
| $q_{vi}$ | 透過通風進入空間的空氣(室外空氣)本身所含有的顯熱 | $q_{vo}$ | 透過通風離開空間的空氣 (室內空氣)本身所含有的顯熱 |
| $q_h$ | 空間中由加熱系統獲得的顯熱 | | |

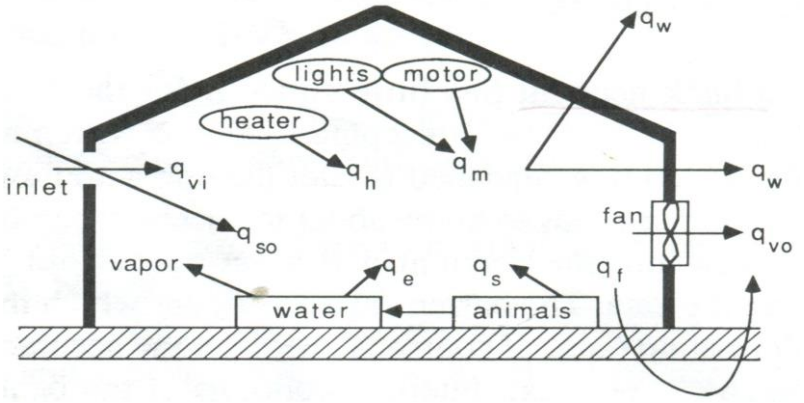



圖 5-1. 顯熱守恆分析的主要構成項

## Gains - Losses = Change of Storage

**Steady state** means no change of storage, thus, **Gains= Losses.**

$$q_s + q_m + q_{so} + q_h + q_{vi} = q_w + q_f + q_e + q_{vo} \text{---------5-1}$$

質量守恆(Mass balance) 分析主要構成項：(圖 5-2)

| | | | |
|---|---|---|---|
| $m_p$ | 空間中由動物體獲得的顯熱 | $m_{vo}$ | 透過通風離開空間的的某成分(水汽、二氧化碳等)的質量 |
| $m_{vi}$ | 透過通風進入空間的某成分(水汽、二氧化碳等)的質量 | $m_p + m_{vi} = m_{vo}$…….. eq. 5-2 | |

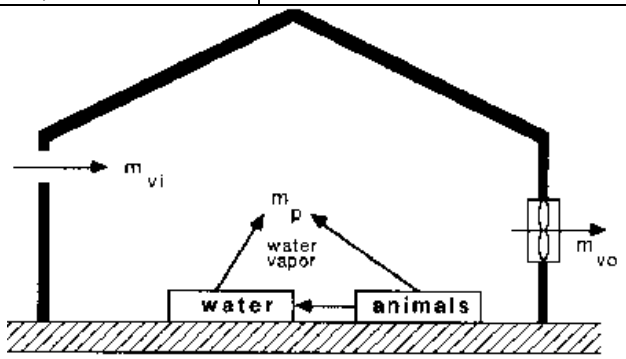



圖 5-2. 質量守恆分析的主要構成項

## 5-2. Components of the Sensible Energy Balance

5-2.1 Sensible heat produced by Animals, $q_s$

- Appendix 5-1. P403

| Animal | Air Temperature | MP mg/kg s | LHP W/kg | SHP W/kg | THP W/kg |
|---|---|---|---|---|---|
| Dairy cow, 500 kg | -1 C | 0.21 | 0.5 | 1.9 | 2.4 |
| | 10 | 0.28 | 0.7 | 1.5 | 2.2 |
| | 15 | 0.36 | 0.9 | 1.2 | 2.1 |
| | 21 | 0.36 | 0.9 | 1.1 | 2.0 |
| | 27 | 0.50 | 1.3 | 0.6 | 1.9 |

- 哺乳類動物的顯熱產生量與體表面積呈線性相關
- 哺乳類動物的顯熱產生量與體重的 0.734 次方成正比

Example 5-1. 試求 100 頭 590 kg 乳牛，處於 12 ºC 室溫下的顯熱產生量。
Determine the sensible heat production from 100 (590kg) dairy cows housed in a barn at 12 C.

$q_{s)500kg} = 1.2 + (2/5)(1.5\text{-}1.2) = 1.32$ W/kg

$Q_{s)500kg} = 1.32 * 500 = 660$ W/cow@500 kg

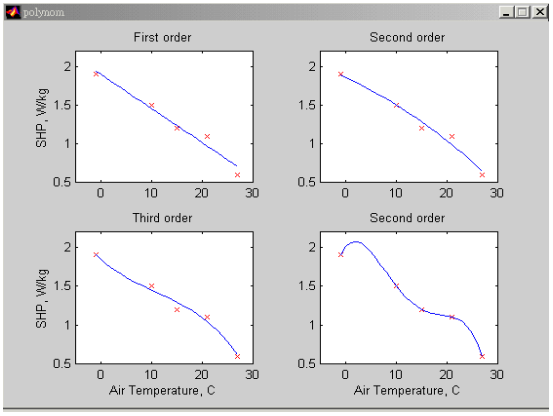
$q_{s)590kg} = 660 * (590/500)^{0.734} = 745$ W/cow@590 kg

$Q_{s)590kg} = 100$ cows $* 745 = 74500$ W $= 74.5$ kW

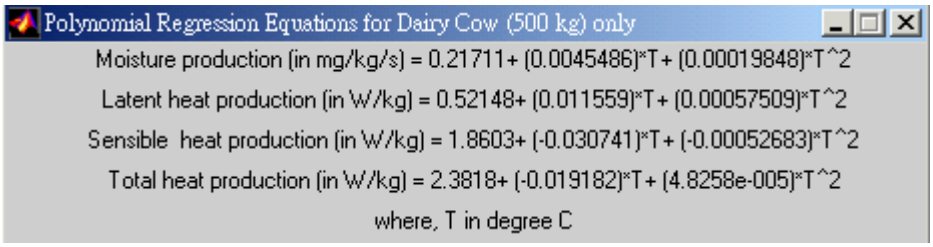
Example 5-2. 請建立二元多項式來求出不同溫度下 500 kg 乳牛的顯熱產生量。可使用 Excel 中工具功能表中的資料分析工具箱中的迴歸功能。以下提供使用 Matlab 撰寫的程式。輸入 polynom，產生一個視窗：學習 Polyfit

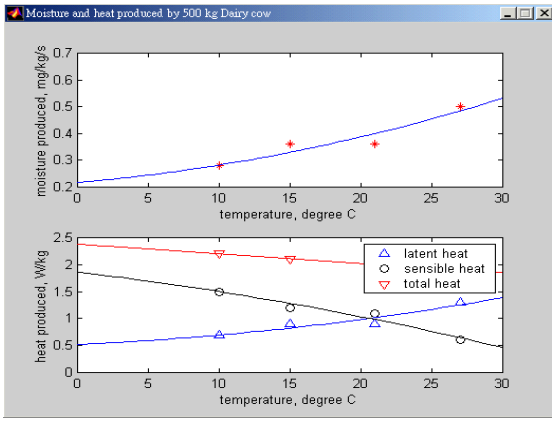
$$q_{s)500\ kg} = 1.86 - 3.074x10^{-2}\ t_{air} - 5.268 \times 10^{-4}\ (t_{air})^2$$

| Observed | Predicted |
|---|---|
| 1.9 W/kg | 1.9 W/kg |
| 1.5 | 1.5 |
| 1.2 | 1.3 |
| 1.1 | 1.0 |
| 0.6 | 0.6 |




輸入 snell_law(1) ，產生兩個視窗：學習 Polyfit 與 fplot 指令




## 5-2.2 Mechanically Produced Heat, $q_m$

Example 5-3. 某禽舍 12 m 寬、40 m 長，使用螢光燈管照明，每平方米地板面積使用 40 W，假設安定器需消耗額外的 20 % 功率。請估算來自燈光的顯熱產生速率。

Sol: 40 * 12 * 40 * 1.2 = 23000 W = 23 kW

## 5-2.3 Solar Heat Gain, $q_{so}$

吸收率 (absorptance, α)、反射率(reflectance, ρ)、穿透率(transmittance, τ)

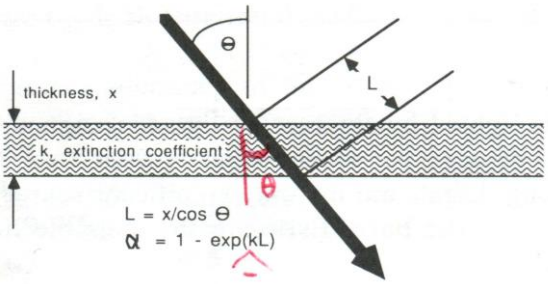
**Absorption**

衰減係數(extinction coefficient, K, in $mm^{-1}$)

穿透距離 (path length, L, in mm)

$-dI_\lambda = I_\lambda K_\lambda dx$ ----------(5-3)

$\alpha = 1 - \exp(-KL)$ ----------(5-4)



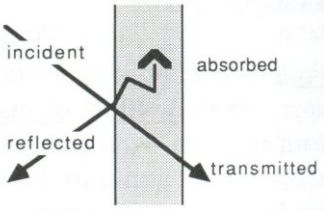




**Table 5-1. Extinction coefficients for transparent materials.**

| Material | Extinction Coefficient, $mm^{-1}$ |
|---|---|
| ordinary window glass | 0.03 (approx.) |
| polyethylene | 0.165 |
| low-iron glass (<0.01%$Fe_2O_3$) | 0.004 (approx.) |
| heat-absorbing glass | 0.13 to 0.27 |
| Tedlar[a] (polyvinyl fluoride) | 0.14 |
| Mylar[a] (polyethylene terephthalate) | 0.205 |
| Teflon[a] (fluorinated ethylene propylene) | 0.06 |

[a]trademark of E. I. DuPont de Nemours, Wilmington, DE.

Example 5-4 請計算通過 3 mm 厚度一般玻璃的垂直入射光線的吸收率

Calculate the solar absorptance at normal incident angle for insolation passing through ordinary window glass 3 mm thick.

Sol: 由表 5-1 可知，衰減係數為 0.03 $mm^{-1}$

由 eq 5-4, $\alpha = 1 - \exp(-KL) = 1 - \exp(-0.03 * 3) = 0.0861$

約 8.6 % 的光線經過玻璃時被吸收了。

**Reflection 反射**

折射率(refractive index)

$n = \sin\varphi / \sin\theta$ ----------(5-5)

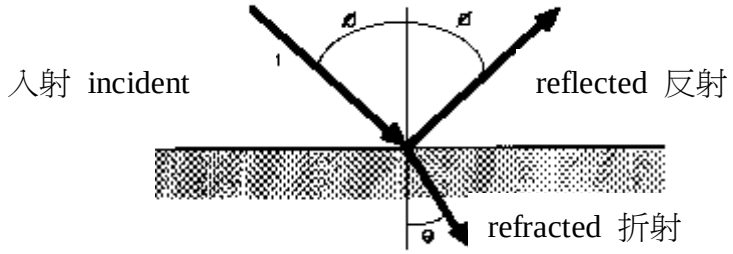



圖 5-3. 光線照射到某透明材料時相關的入射、反射與折射角

Snell's law:

$$\rho_{parallel} = \sin^2(\varphi-\theta)/\sin^2(\varphi+\theta) \text{----------(5-6a)}$$

$$\rho_{perpendicular} = \tan^2(\varphi-\theta)/\tan^2(\varphi+\theta) \text{----------(5-6b)}$$

Example 5-5. 光線入射玻璃時入射角為 50 度，請計算水平與垂直方向的折射率。
Determine the two components (parallel and perpendicular) of reflectance of light irradiating a glass surface at an angle of 50 degrees.

$\theta$ = arcsin (sin 50°/1. 50) = 30.7°

$\rho_{parallel} = \sin^2(50 - 30.7) / \sin^2(50 + 30.7) = 0.112$

$\rho_{perpendicular} = \tan^2(50 - 30.7) / \tan^2(50 + 30.7) = 0.003$

$\rho_{average} = (0.112 + 0.003) / 2 = 0.058$

輸入 snell_law(2)，直接計算不同入射角下的反射率。產生視窗如下：

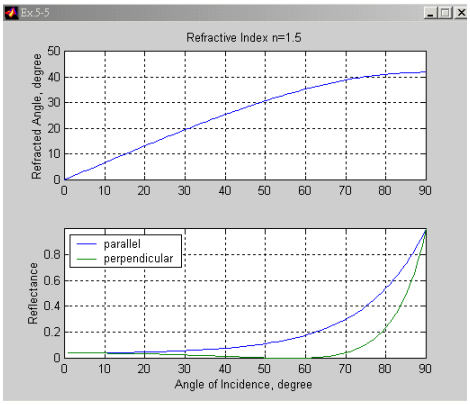



**Table 5-2. Refractive indices for light in the visible waveband.**

| Material | Refractive Index |
|---|---|
| air | 1.00 |
| window glass | 1.50 to 1.55 |
| Tedlar[a] (polyvinyl fluoride) | 1.45 |
| Mylar[a] (polyethylene terephthalate) | 1.64 |
| Teflon[a] (fluorinated ethylene propylene) | 1.34 |

[a]trademark of E.I. DuPont de Nemours, Willmington, DE.

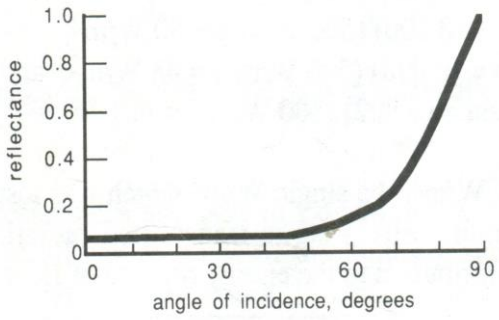



**Figure 5-4. Reflectance (specular) for light incident on glass as a function of the angle of incidence.**

**Transmittance 穿透率**

Stokes' equations

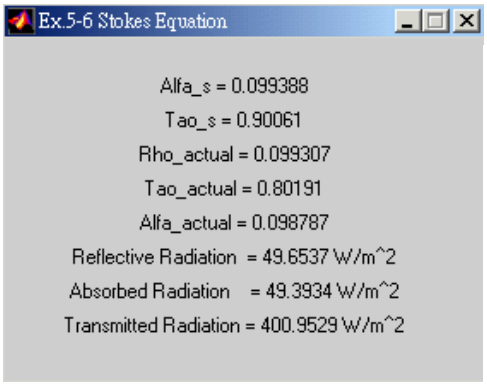
$$\rho_{actual}=\rho_s * (1+\tau_s*\tau_{actual})\text{----------(5-7a)}$$

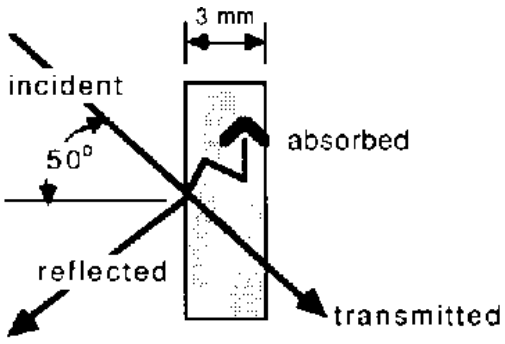
$$\tau_{actual}=\tau_s *(1-\rho_s)^2 / (1-\rho_s^2\tau_s^2)\text{----------(5-7b)}$$

$$\alpha_{actual} = 1 - \tau_{actual} - \rho_{actual} = (1-\rho_s)( 1-\tau_s)/(1-\rho_s\tau_s)\text{----------(5-7c)}$$

Example 5-6. 入射角為 50 度，請計算太陽能為 500 W/m$^2$ 的光線經過 3 mm 一般玻璃之後有多少被吸收、穿透與反射? 吸收率、穿透率與反射率各為何?

輸入 snell_law(3) ，產生視窗如下：




L = 3 mm/cos (30.7°) = 3.49 mm

$\alpha_s$ = 1 – exp (- 0.03 mm$^{-1}$*3.49 mm) = 0.099

$\tau_s$ = 1.0 – 0.099 = 0.901

$$\rho_{actual} = 0.058(1+\frac{0.901^2(1-0.058)^2}{1-0.058^2\ 0.901^2} = 0.100$$

$$\tau_{actual} = 0.901(\frac{(1-0.058)^2}{1-0.058^2\ 0.901^2}) = 0.802$$

$\alpha_{actual}$ = 1.0 – 0.100 – 0.802 = 0.098

5-2.4. Heating System, $q_h$

5-2.5. Ventilation, $q_{vi}$ and $q_{vo}$

$$q_{vo} - q_{vi} = 1006*\rho\dot{V}\ (t_i-t_o) \text{ ----------(5-8)}$$

5-2.6. Structural Heat Loss, $q_w$

$$q_w = \Sigma\ (A/R)_n*(t_i-t_o) \text{ ----------(5-9)}$$

5-2.7. Heat exchange with the floor, $q_f$

$$q_f = F*P*(t_i-t_o)$$

5-2.8. Evaporation, $q_e$

| 結構物 | 水分來源 |
|---|---|
| barns | wash water, animal wastes and animal's respiration |
| greenhouses | floors and benches when water is spilled, from the surface of the potting medium, and from transpiration by plants. |

As a rough rule, insolation which passes through the greenhouse cover can be partitioned into 5 categories.

| | |
|---|---|
| 50 % | Converted to sensible heat added to the air |
| 25 % | Added to the air as latent heat |
| 10 % | Reflected back to the outside |
| 2-3 % | Used in photosynthesis |
| 12 - 13 % | Stored in the intrinsic thermal mass to be released later |

## 5-3. Uses of the Sensible Energy Balance

$$q_s + q_m + q_{so} + q_h + q_{vi} = q_w + q_f + q_e + q_{vo} \text{ ----------(5-1)}$$

$$q_s + q_m + q_{so} + q_h = \Sigma\ UA(t_i-t_o) + FP(t_i-t_o) + q_e + 1006\rho\dot{V}\ (t_i-t_o) \text{ --(5-10)}$$

用於 Animal housing，上式中有多項可簡化或取消：

$$q_s = (\Sigma\ UA + FP + 1006\rho\dot{V}\ )(t_i-t_o) \text{ ----------(5-11)}$$

$$t_i = t_o + q_s /(\Sigma\ UA + FP + 1006\rho\dot{V}\ ) \text{ ----------(5-12)}$$

$$\dot{V} = [q_s - (\Sigma\ UA + FP)\ (t_i-t_o)] / [1006\rho(t_i-t_o)] \text{ ----------(5-13)}$$

Example 5-7 -請計算在以下條件下所需要的通風風量率 ($m^3/s$) 以維持乳牛舍於 15 度 C。海拔 500 m，60 頭平均 580 kg 乳牛，牆、天花板、窗戶、門之面積依序為 274、520, 12, 15 $m^2$，熱阻依序為 2.05, 1.97, 0.3, 0.49 $m^2K/W$，周長 110 m，周長熱損因子 1.5 W/mK，室外溫度 -5 度 C，室內濕度 70%。假設燈光、馬達與透過窗戶進入室內的能量均可以忽略不計算。

The attic is well ventilated. The animal heat data for fairy cows in Appendix 5-1 reflect net sensible heat after latent heat conversion has been deducted.

Sol: $\sum U A = \sum A/R = 274/2.05 + 520/1.97 + 12/0.3 + 15/0.49 = 470$ W/K

$F*P = 15 * 110 = 165$ W/K

$q_{s)500kg} = 1.2$ W/kg at 15 degree C

$Qs = 60 * 1.2 * 500 * (580/500)^{0.734} = 40000$ W = 40 kW

From eq. 5-13, Ventilation rate = $[40000 – (470 + 165)*20] / [1006*1.14 * 20]$

$= 1.2$ $m^3/s = 1.14$ $kg/m^3 * 1.2$ $m^3/s = 1.37$ kg/s

Example 5-8. 續上題，假設最低需維持通風風量率 0.9 $m^3/s$ 以維持動物體健康，請問室外的最低溫是多少時，室內將無法維持 15 度 C。

$$t_o = t_i - q_s /(\Sigma UA + FP + 1006\rho\dot{V}) \text{ ----------(5-12b)}$$

$= 15 – 40000 / (470 + 165 + 1006*1.14*0.9) = -9$ deg.C

Example 5-9. 續上題，假設通風風量率為 2 $m^3/s$，室內溫度會變成幾度 C？

$t_i$ 未知，$q_s$ 無法求，需透過疊代計算

假設室內空氣密度為 1.15 $kg/m^3$，動物體顯熱產生量為 **45 kW**

$t_i = -5 + 45000 / [470+165+1006*1.15*2] = 10.3$ deg.C

Now check the assumption

At indoor T = 10.3 deg.C, $q'_s = 1.5 + (0.3/5)(1.2-1.5) = 1.48$ W/kg

$q_s = 60 * 1.48 * 500 * (580/500)^{0.734} = 49500$ W = **49.5 kW**

此數字比假設值大，顯然假設有誤，空氣密度與顯熱產生量都低估了

重新假設室內空氣密度為 1.16 $kg/m^3$，動物體顯熱產生量為 **48 kW**

$t_i = -5 + 48000 / [470+165+1006*1.15*2] = 11.2$ deg.C

Now check the assumption

At indoor T = 11.2 deg.C, $q'_s = 1.5 + (1.2/5)(1.2-1.5) = 1.4$ W/kg

$q_s = 60 * 1.4 * 500 * (580/500)^{0.734} = 47000$ W = **47 kW**

此數字比假設值小，顯然假設有誤，空氣密度與顯熱產生量都高估了

經幾次疊代運算可求出室內溫度為 11 度 C，

室內空氣密度為 1.16 $kg/m^3$，動物體顯熱產生量為 **47.5 kW**

## 5-4. Components of the Mass Balance, Humidity

5-4.1. Moisture Production From Animals

Example 5-10. 某豬舍內有 40, 60, 80 公斤的三種豬各 100 頭，室溫維持在 20 度 C，請問在單位時間內需移除多少水分才能維持穩態。(Appendix 5-1. p.403)

| Size, kg | Moisture Production mg/kg-s | mg/pig-s |
|---|---|---|
| 40 | 0.61 | 24 |
| 60 | 0.47 | 28 |
| 80 | 0.39 | 31 |

100 * (24+28+31) / $10^6$ = 0.0083 kg/s

Example 5-11. 牛舍內有 100 頭 570 kg 的乳牛，請計算在 10 度 C 室溫下的水分產生率，這同時也是需被移除的速率以維持穩態？

| Animal | Air Temperature | MP mg/kg s | LHP W/kg | SHP W/kg | THP W/kg |
|---|---|---|---|---|---|
| Dairy cow, 500 kg | -1 C | 0.21 | 0.5 | 1.9 | 2.4 |
| | 10 | 0.28 | 0.7 | 1.5 | 2.2 |
| | 15 | 0.36 | 0.9 | 1.2 | 2.1 |
| | 21 | 0.36 | 0.9 | 1.1 | 2.0 |
| | 27 | 0.50 | 1.3 | 0.6 | 1.9 |

Moisture production rate = 0.28 * $(570/500)^{0.734}$ = 0.31 mg/kg-s

$m_p$ = 0.31 mg/kg-s (570 kg)(100 cows)(1.0 x $10^{-6}$ kg/mg) = 0.0157 kg/s

5-4.2. Ventilation, $m_{vi}$ and $m_{vo}$

$$m_{vi} = \rho_o \dot{V}_o W_o \text{----------(5-14)}$$

$$m_{vo} = \rho_i \dot{V}_i W_i \text{----------(5-15)}$$

## 5-5. Uses of the Mass Balance, Moisture

$$m_{air} = m_{water} / (W_i - W_o) \text{----------(5-16)}$$

Example 5-12. 某雞舍位於海拔 1000 m，飼養 3 萬頭平均體重 1.8 kg 的來亨蛋雞，當室外溫、濕度為-20 ºC、55 %，欲維持室內為 23 ºC、70 % 相對濕度時的通風風量率需要多少？

| Animal | Air Temperature | MP mg/kg s | LHP W/kg | SHP W/kg | THP W/kg |
|---|---|---|---|---|---|
| Laying hen, leghorn, 1.8 kg (a) | 8 | 0.72 | 1.7 | 5.2 | 6.9 |
| | 12 | 0.82 | 2.0 | 4.6 | 6.6 |
| | 18 | 0.97 | 2.3 | 4.5 | 6.8 |
| | 28 | 1.19 | 2.9 | 3.7 | 6.6 |

At 23 ºC indoor, MP = (0.97 + 1.19)/2 = 1.08 mg/kg.s

$m_{water}$ = 30000 * 1.8 * 1.08 = 58320 mg/s = 0.05832 kg/s

Wo = 0.000393 kg/kg DA at -20 ºC and 55% RH outdoor

Wi = 0.013919 kg/kg DA at 23 ºC and 70 % RH indoor

From eq. 5-16,

$m_{air}$ = (0.05832)/(0.013919 – 0.000393) = 4.3 kg/s (Dry Air)

$m_{air}$ = 4.3 * (1 + 0.013919 ) = 4.36 kg/s (Moist Air)

$V_{air}$ = 4.36 kg/s / 1.03 kg/$m^3$ = 4.23 $m^3$/s

## 5-6. Components of the Mass Balance, Carbon Dioxide

5-6.1. Carbon Dioxide produced By Animals

1 L of CO2 is produced, on the average, for every 24.6 kJ of total heat added to the environment by an animal. (1 L/s $CO_2$ = 24.6 kW)

## 5-7. Uses of the Mass Balance, Carbon Dioxide

大氣中的二氧化碳濃度約在 345 ppm (parts per million, 體積比例)，相當於 345 x 1.519 = 524 mg of $CO_2$/kg of Air.

Example 5-13. 續上例，計算風量率為 4.3 $m^3$/s 下的室內 $CO_2$ 含量。

$V_p + V_{vi} = V_{vo}$

$q_{total}$ = 30000 * 1.8 * (6.6+6.8)/2 = 361800 W = 361.8 kW

Vp = 361.8 / 24.6 = 14.7 L/s = 0.0147 $m^3$/s

0.0147 + [4.3 $m^3$/s * 0.000345 ] = [4.3 $m^3$/s * $CO_{2indoor}$]

$CO_{2indoor}$ = 0.003764 = 3764 ppm

Example 5-14. 海拔為 0 m 體積為 3000 $m^3$ 的溫室，因密閉不良導致的換氣率為 0.75 ACH (air change per hour)，假設室內空氣溫度 20 ºC，相對濕度 70 %，溫室外 $CO_2$ 濃度為 345 ppm，溫室內補充 $CO_2$ 濃度到 1000 ppm (1519 mg/kg) 以提升植物生長速率，請問 $CO_2$ 的損失速率有多少 kg/s？假設 1 kg $CO_2$ 為 20 元，每天會損失多少錢?

Sol:

$T_{air}$@ 20 ºC, $RH_{air}$@ 70 % → 空氣密度 1.18 $kg/m^3$

$m_{air}$ = 1.18 $kg/m^3$ * 3000 * 0.75 /3600 = 0.7375 kg/s

$m_{vo}$ = 0.7375 kg/s * 1.519 * (1000 – 345) = 733.8 mg/s = 2.64 kg/hr

損失略低於 2.64 * 24 * 20 = 1267 元，因為暗期的呼吸作用會補回一些。

# **Chap. 5** 能量與質量守衡之穩態分析

Home Work

1. Text Book ex 5-3 on p171 of Textbook. (使用 POLYNOM 程式)
2. Text Book ex 5-5.
3. Text Book ex 5-6.
4. Text Book ex 5-7.
5. Text Book ex 5-8.
6. 沿用 p165-166 example 5-12 的基本數據，分別計算要維持室內狀況為 23℃或 70%RH 時所需之通風率 (ventilation rate)，並比較二者之大小。假設戶外溫度為 10, 15, 20, 25, 30℃，濕度為 50, 75, 90%。當外溫大於 23℃時會啟動水牆，假設水牆效率為 75%，請完成下表。(注意在某些溫濕度狀況下將無法達到需求的室內狀況，請註明"無法達成"，當此情況發生，改計算需除去多少熱或多少水份才能達到目標，使用另表說明)

| Required Ventilation rate | 相對濕度 50% | | | | |
|---|---|---|---|---|---|
| | 外溫 10 | 外溫 15 | 外溫 20 | 外溫 25 | 外溫 30 |
| a.溫度控制 23℃ | | | | | |
| b.濕度控制 70% | | | | | |
| Max (a, b at 50%) | | | | | |
| Required Ventilation rate | 相對濕度 75% | | | | |
| | 外溫 10 | 外溫 15 | 外溫 20 | 外溫 25 | 外溫 30 |
| a.溫度控制 23℃ | | | | | |
| b.濕度控制 70% | | | | | |
| Max (a, b at 75%) | | | | | |
| Required Ventilation rate | 相對濕度 90% | | | | |
| | 外溫 10 | 外溫 15 | 外溫 20 | 外溫 25 | 外溫 30 |
| a.溫度控制 23℃ | | | | | |
| b.濕度控制 70% | | | | | |
| Max (a, b at 90%) | | | | | |

7. 將上題所完成的表格中的 Max(a,b)值 針對外溫(10-30℃)予以繪圖，隨濕度不同應有三條曲線。
8. Text Book ex 5-9 改用 Matlab 撰寫程式 (直接解 Ti).
9. Text Book ex 5-12 改用 Matlab 撰寫程式 (使用 psy.m 中的 functions 求解 Humidity ratio).